REPORT 1043

A NUMERICAL METHOD FOR THE STRESS ANALYSIS OF STIFFENED-SHELL STRUCTURES UNDER NONUNIFORM TEMPERATURE DISTRIBUTIONS [1]

By Richard R. Heldenfels

SUMMARY

A numerical method is presented for the stress analysis of stiffened-shell structures of arbitrary cross section under nonuniform temperature distributions. The method is based on a previously published procedure that is extended to include temperature effects and multicell construction. The application of the method to practical problems is discussed, and an illustrative analysis is presented of a two-cell box beam under the combined action of vertical loads and a nonuniform temperature distribution.

INTRODUCTION

The effects of nonuniform temperature distributions, such as those produced by aerodynamic heating, are becoming of greater concern in the design of modern high-speed aircraft. The structural effects of temperature changes and the results of some analyses of a simplified structure under nonuniform distributions of temperature have been discussed in reference 1. The analytical methods considered in reference 1 were found, however, to yield inaccurate values for the secondary stresses in complicated structures, and in such cases some type of numerical approach is desirable. Numerical methods, however, usually require extensive and tedious calculations and they should be used only when satisfactory results cannot be obtained from a simplified analysis.

Several numerical methods of stress analysis have been presented in the literature, but none contains provisions for temperature changes. In the present report, one such method, the numerical procedure of reference 2, has been extended to include the effects of a nonuniform distribution of temperature. In addition, the equations developed permit the analysis of a stiffened-shell structure of arbitrary cross section with any number of internal cells. The application of the method is discussed and illustrated by analysis of a two-cell box beam under the combined action of vertical loads and a nonuniform temperature distribution.

DESCRIPTION OF THE NUMERICAL METHOD

BASIC THEORY

The structure analyzed is an idealized representation of a multicell stiffened-shell structure (see fig. 1) and has the following characteristics:

(1) The basic unit is a rectangular panel bounded on two parallel sides by extensionally flexible stringers and on the other two sides by rigid bulkheads.

(2) The panels consist of sheet material and are assumed to carry shear stress only. The shear stress is constant within a given panel.

(3) The stringers run parallel to the direction of the primary stresses and carry axial load only.

(4) The bulkheads lie perpendicular to the stringers and are rigid in their own plane but offer no resistance to warping out of their plane.

(5) The structure is loaded only at the bulkheads.

(6) Material properties, cross-sectional dimensions, and temperature distribution do not vary along the length of a given bay.

With these assumptions about the basic elements of the structure, any type of stiffened shell can be analyzed, provided taper is excluded. The state of stress in such a structure can then be defined by suitable stress-strain relations and two types of displacements:

(1) Stringer displacements, which are displacements, at the end of a bay, of each flexible stringer in a direction parallel to the stringer

(2) Bay displacements, which are translations and rotations of the plane of each cross section defined by the rigid bulkheads

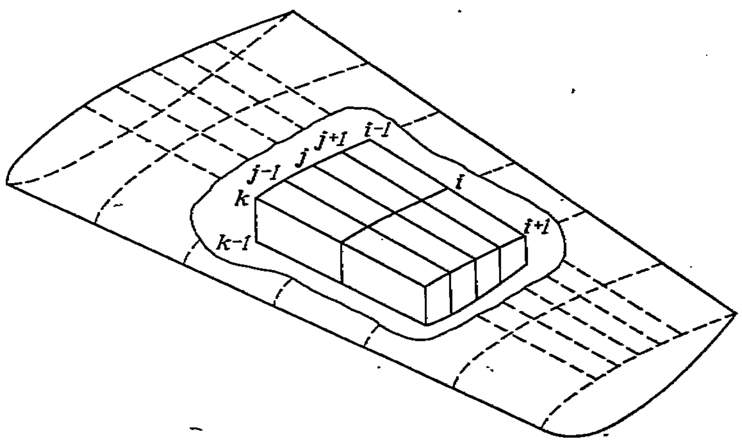


FIGURE 1.—Typical multicell, stiffened-shell wing structure.

[1] Supersedes NACA TN 2241, "A Numerical Method for the Stress Analysis of Stiffened-Shell Structures under Nonuniform Temperature Distributions" by Richard R. Heldenfels, 1950.

Once the stress-strain relations are established for the components of the idealized structure, equations of equilibrium can be used to obtain relationships between the displacements. The equilibrium equation for the forces on an individual stringer yields an expression for the stringer displacement of any panel point in terms of the surrounding stringer displacements and the displacements of the two adjacent bays. From this general expression, equations equal in number to the unknown stringer displacements are obtained. The additional equations required for the determination of the bay displacements are obtained from the equations of equilibrium of the shear forces on the cross sections. These equations then completely define the displacements of the structure. In most cases the number of equations is so large that a direct solution would be impractical and it has been found expedient to solve them by the recommended iteration procedure described in the next section. The required equations are derived in detail in the appendix.

SOLUTION OF EQUATIONS BY ITERATION

Matrix iteration often provides the easiest and quickest solution to the equations, and the procedure recommended is as follows:

The equations to be solved can be written in matrix notation as

$$[B]\{d\}=\{c\} \tag{1}$$

For purposes of iteration, these equations are rearranged to give

$$\{d\}=[C]\{d\}+\{c\} \tag{2}$$

where

$[C]=[U]-[B]$

$[B]$ square matrix of coefficients of general equations with diagonal terms reduced to unity

$[U]$ unit matrix

$\{d\}$ column matrix of stringer and bay displacements

$\{c\}$ column matrix of constant terms in general equation; these terms arise from applied load and thermal expansion

Initial approximate values of stringer and bay displacements $\{d_0\}$ are then selected. These values may be determined in any convenient manner; however, subsequent operations can be simplified, as explained in the appendix, if these values are chosen to correspond to elementary theory. Next, the initial displacement values are substituted into the righthand side of equation (2) to obtain a second approximation $\{d_1\}$ to the displacements

$$\{d_1\}=[C]\{d_0\}+\{c\} \tag{3}$$

and the differences between the second and initial approximate displacement values are computed from the equation

$$\{\Delta d_1\}=\{d_1\}-\{d_0\} \tag{4}$$

The iteration process is then begun by using these displacement differences. The nth difference is defined as

$$\{\Delta d_n\}=\{d_n\}-\{d_0\} \tag{5}$$

and it can be easily verified that the use of these differences leads to the following matrix equation:

$$\{\Delta d_n\}=[C]\{\Delta d_{n-1}\}+\{\Delta d_1\} \tag{6}$$

The iteration process consists of a series of solutions of equation (6), each successive solution yielding a better approximation to the displacement differences than the previous one. The process is continued until successive solutions of equation (6) yield the same result, that is, until

$$\{\Delta d_n\}=\{\Delta d_{n-1}\} \tag{7}$$

The final displacements are then determined from the final differences by using equation (5) and the initial values.

When equation (6) is being iterated, improved values should be used as soon as they are obtained; that is, each individual difference Δd_n should be substituted into the $\{\Delta d_{n-1}\}$ matrix immediately after calculation rather than at the end of the cycle. In this manner, each new value determined receives the benefit of all previous work and convergence is speeded.

The iteration of differences reduces the work required to obtain a solution because smaller numbers are involved. However, it is essential that no errors be made in the determination of the first differences $\{\Delta d_1\}$ since a single significant error will render the whole solution useless.

CONVERGENCE OF THE ITERATION PROCESS

In order to obtain more rapid convergence of the iteration process, bay displacements and loads are referred to the principal shear axes of each bay. The use of these axes greatly simplifies the equations for bay displacements by making each bay displacement independent of all other bay displacements and thus a function of the stringer displacements only. In addition, a special correction cycle is periodically introduced to bring the stringer forces on each cross section into equilibrium with the applied loads. Mathematically, the correction cycle is a special cycle that uses a certain combination of the basic equations. Its success in the particular case of the numerical method of stress analysis is a result of its physical significance, and in that respect it is similar to Southwell's "group relaxations" (reference 3).

The optimum frequency of application of the correction cycle depends largely on the characteristics of each individual problem and must be determined on a basis of experience with the method. If this frequency cannot be determined from previous experience, it can be approximated satisfactorily by one that permits the disturbances to spread their significant effect over the structure between correction cycles.

The application of the correction cycle begins at a station where the displacements are known and then proceeds outboard. The corrections required to bring the first bay into equilibrium are determined, and the stringer displacements at its outboard end are changed accordingly before the corrections required by the second bay are calculated.

EFFECT OF INTRODUCING NONUNIFORM TEMPERATURE DISTRIBUTIONS

The preceding method is applicable to any type of stress problem. Nonuniform temperature distributions do not affect the general procedure but merely change the details. These effects are of two types: A change in the effective structure due to changes in elastic properties of the material with temperature and thermal stresses resulting from restrained thermal expansion. The changes in elastic properties are easily handled if the moduli are treated as variables during the derivation of the equations. Their effect is analogous to that of variations in stringer area and panel thickness. The presence of thermal expansion requires modification of the stress-strain relationships for the stringers but does not affect those for the panels. The equations for stringer displacements contain thermal-expansion terms that are analogous to the applied-load terms. Bay-displacement equations are unaffected by thermal expansion, but thermal-expansion terms appear in the equations used for the correction cycle. If a difference solution is iterated, the elementary solution should include the distributions of thermal strain associated with the primary thermal stresses, which may be obtained from the equations derived in the appendix.

DISCUSSION OF THE NUMERICAL METHOD

The application of a method, such as that just described, always poses a number of questions; for example, what are some of the limitations of the method, would it be advantageous to use some other method of analysis, and how should the structure be idealized? Some of the factors requiring consideration, other than those mentioned in the previous section, are therefore now discussed.

VALIDITY OF BASIC ASSUMPTIONS

The assumptions upon which the method is based are commonly accepted in the analysis of stiffened shells. Comparison of theoretical and experimental results has established the fact that these assumptions will yield good results in most cases. Two important assumptions—that the bulkheads are rigid in their own plane and that the shear stress is constant in a given panel—may, however, introduce significant errors into the analysis in some cases. These assumptions are therefore examined in detail.

The assumption of rigid bulkheads is satisfactory as long as the primary stresses run perpendicular to the bulkheads, but, as demonstrated in reference 1, this assumption may not be good when dealing with problems involving thermal stress. Large temperature gradients along the length of the structure or across the depth of a bulkhead distort the real bulkhead and make the assumption of rigidity inapplicable. In many cases, however, these effects are small and the assumption yields satisfactory results.

The numerical method could be extended to include the effects of bulkhead flexibility. Such an analysis, however, is very cumbersome and tedious and, if the equations are solved by iteration, the process is often very slowly convergent. Therefore, these extensions are not considered herein.

The assumption of constant shear stress in a given panel simplifies the development of the equations, and it yields satisfactory results if the bulkheads are reasonably close together. Cases arise, however, in which the assumption will lead to unreasonable results because the assumed constant shear stress is a poor approximation to a shear stress which should be changing rapidly in the spanwise direction. This situation is usually accompanied by slow convergence of the iteration process. This difficulty, however, can be minimized by reducing the bulkhead spacing of the idealized structure since it occurs only when the total shear stiffness of the panels joined to a stringer exceeds the extensional stiffness of that stringer.

IDEALIZATION OF AN ACTUAL STRUCTURE

The idealization process described in reference 2 is straightforward. However, it provides an opportunity for the stress analyst to exercise his engineering judgment and thus to simplify the analysis. By restricting the analysis to only a part of the structure or by using a comparatively simple idealized structure, the time required for the analysis can be substantially reduced. Such simplifications, however, can reduce the value of the results, and a compromise between speed and exactness is required.

The number and location of the idealized stringers completely define the stress-distribution shapes obtainable from the analysis. (For example, in an idealized shell of n stringers, there are n possible types of independent normal-stress distributions, three of which can be determined from elementary theory, the remaining $n-3$ being statically indeterminate.) Stringer location is thus an important part of the idealization process and in conventional problems the locations should be selected after consideration of the characteristics of the actual structure, the nature of the expected results, and the time available for the analysis. When nonuniform temperature distributions are involved, the shape of the temperature distribution should also be considered because the thermal-stress and temperature distributions will have similar shapes and the analysis will yield good results only if the idealized structure permits a stress distribution of that shape.

The bulkhead spacing usually is the same in the idealized and actual structures, but the idealized spacing should never be so large that trouble is caused by the assumption of constant shear stress. A proper bulkhead spacing is one for which the extensional stiffness of each stringer element is greater than the sum of the shear stiffnesses of the adjacent panels so that no negative terms appear on the right-hand side of the equations for the stringer displacements.

CALCULATING PROCEDURES

All the calculations required by the numerical procedure (determining the coefficients of the equations, solving by iteration, and computing the stresses) are routine and involve only simple arithmetic. The calculations can be easily arranged in tabular form so that the bulk of the work can be done by modern automatic computing machinery or by a computer who does not need to have a knowledge of the structural theories involved.

In any problem that involves extensive numerical work, errors are very apt to occur. One of the advantages of the numerical method described herein is that a number of checking procedures can be devised to check the various steps in the calculations. No attempt is made to describe the many possible checks; a few, however, have been indicated in the illustrative example.

Solution of the equations by simple iteration also possesses another advantage with regard to errors. Values obtained from successive cycles of iteration show trends that can be observed by an experienced computer, and errors can be detected by their effects on these trends. Errors that do appear during the iteration process eventually work themselves out but may adversely affect the rate of convergence.

APPLICATION OF THE NUMERICAL METHOD

DESCRIPTION OF THE PROBLEM

The application of the numerical method is illustrated by an analysis of the idealized two-cell box beam shown in figure 2. The cross section is symmetrical about the horizontal center line and the beam is untapered; however, the stringer areas and sheet thicknesses vary from bay to bay.

The box beam is loaded by four concentrated vertical loads applied at the bulkhead stations along the inner web; in addition, it is subjected to the arbitrarily selected distribution of temperature increase shown in figure 3. The temperature is highest at the tip and along the front web and decreases in both spanwise and chordwise directions, but it is constant across the depth of the beam. The beam is assumed to be constructed of 75S–T6 aluminum alloy which has the variation of elastic properties with temperature increase shown in figure 4. These data are the same as those used in reference 1.

It is assumed that no thermal stresses were present at 60° F. Since the method of analysis involves the assumption that no changes in temperature distribution occur over each element, the temperature used in the calculations was the temperature at the center of the element concerned.

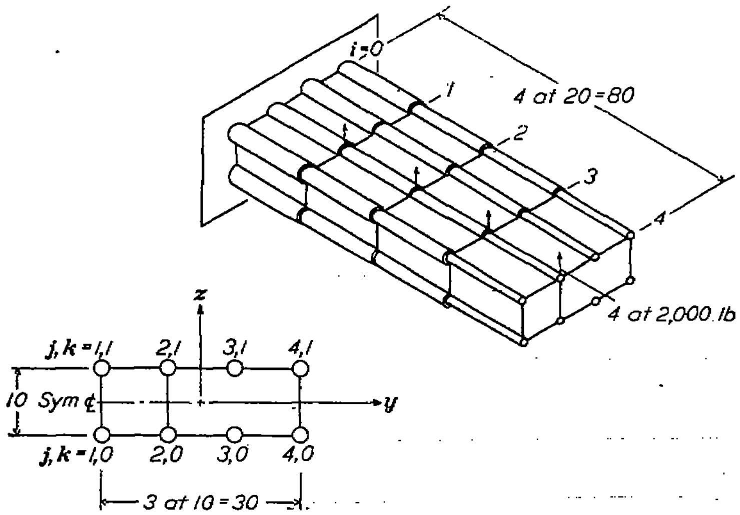


FIGURE 2.—Idealized structure used in illustrative example. (Stringer areas and skin thicknesses are listed in tables I and II.)

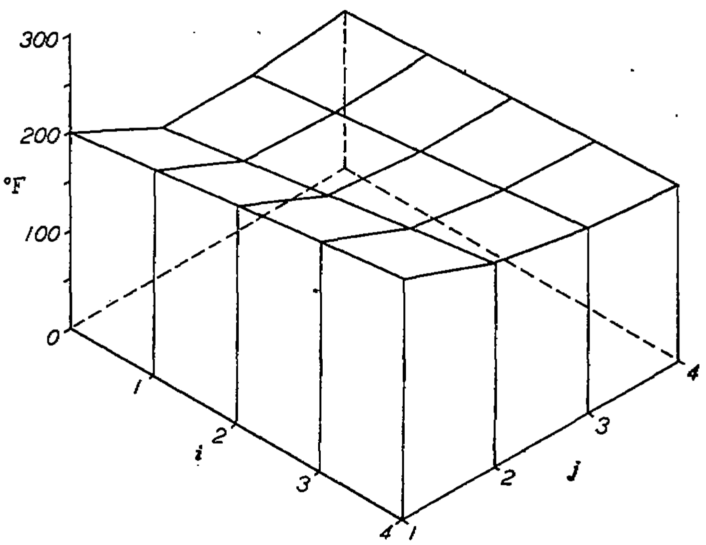


FIGURE 3.—Distribution of temperature increase.

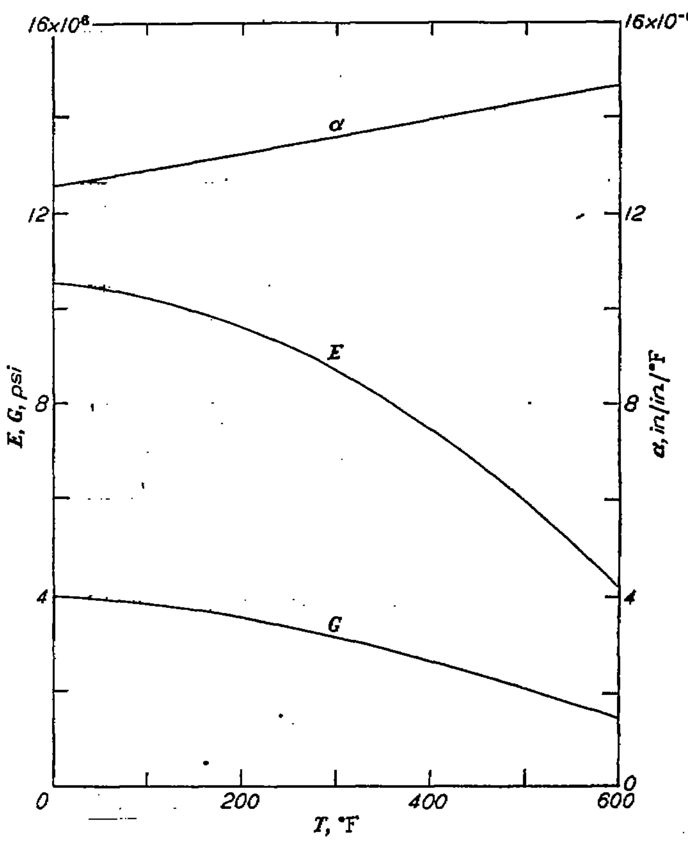


FIGURE 4.—Variation of elastic properties of 75S–T6 aluminum alloy with temperature increase (reference 1).

DETAILS OF THE ANALYSIS

Since the structure and the temperature distribution are symmetrical about the horizontal center line, the analysis can be restricted to one cover. Two solutions are required however, to determine the total stress since the thermal-stress system is symmetrical about the horizontal center line, but the load-stress system is antisymmetrical. In this way, the analysis requires the solution of two sets of equations (one of 20, the other of 24) which can be solved more easily than the set of 44 needed for a single analysis of the complete box.

The computations required are given in tabular form with most tables containing two parts—one related to the load stresses and the other related to the thermal stresses. The final solution is obtained by the superposition of these two solutions. The rectangular cross section and its symmetry permit several simplifications of the general equations. In each case the equations used are listed. The notation is described in the appendix. Methods used to check the calculations are also given in the tables. The checking methods used were determined from mathematical relationships existing between the coefficients of the equations and from equilibrium of forces.

Tables I and II present the physical characteristics and stiffness parameters of the individual stringers and panels. Table III gives the location of the principal shear axes of each bay and the coefficients of the bay-displacement equations. The location of the principal inertia axes of each bay, the coefficients used in the correction cycle, and the initial stringer displacements are given in table IV. The coefficients of the stringer-displacement equations are tabulated in table V. Table VI contains the $[C]$ matrices used for the iteration. The rows and columns have been interchanged in order that the matrix multiplications required will consist of the cumulative multiplication of the adjacent numbers in two columns. Table VII is a similar arrangement of data required for the correction cycle and also lists each correction determined. The displacements obtained from each cycle of iteration are given in table VIII and the correction cycles are indicated. Table IX contains the calculation of each type of stress and the superposition required to obtain the total stresses.

The numerical calculations in this example were done by a computer who had previous experience with the method. The following times were required:

Setting up the equations (table I to table VII)	3 days
Solving the equations (table VIII)	4 days
Computing stresses (table IX)	1 day

In this example, the displacements were computed to six decimal places (five or six significant figures) in order that the stress would be accurate to 1 psi and thus would provide accurate equilibrium checks. Most practical problems will not require such numerical accuracy and a smaller number of decimal places should be used in order to speed the solution. It is estimated that the time required to solve this example could have been reduced by one-half if the number of decimal places had been reduced from six to four. This reduction would have given stresses accurate to 100 psi or about 1 percent of the maximum stress.

RESULTS OF THE CALCULATION

The results of the calculation are shown graphically in figure 5 by spanwise and chordwise plots of the stringer stresses in the top and bottom covers and a spanwise plot of shear stresses in the webs. The spanwise plots have a jagged appearance because stringer areas and sheet thicknesses are assumed constant in each bay with an abrupt change at the bulkheads. The dashed lines in the plots of stringer stresses are the values obtained from an elementary analysis.

CONCLUDING REMARKS

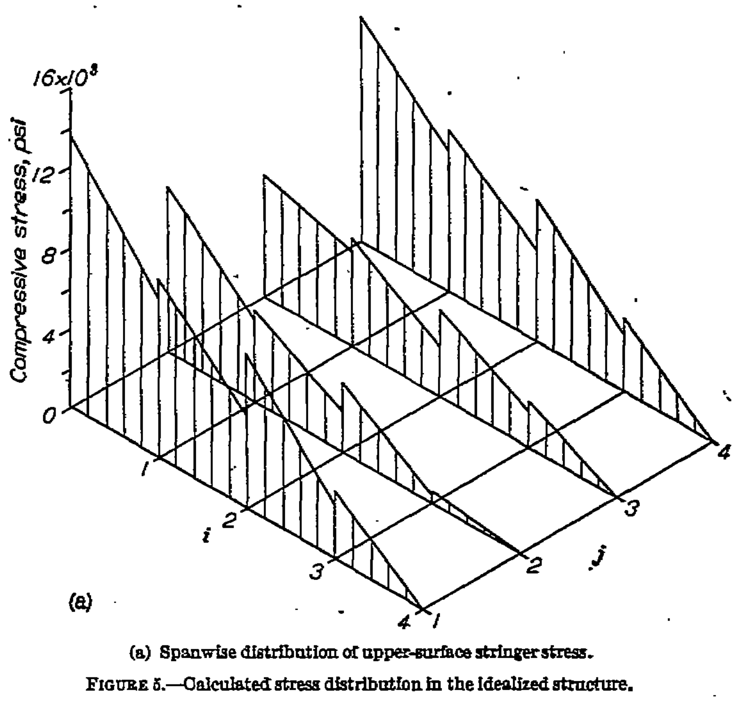
A numerical method for the stress analysis of stiffened-shell structures under nonuniform temperature distributions has been presented. The method is not applicable to the solution of all structural problems involving temperature effects because it requires extensive and tedious calculations and because the basic assumptions of bulkheads rigid in their own plane and constant shear stress in a given panel occasionally lead to unsatisfactory results. It is, however, a powerful tool for the solution of many structural problems because:

(1) It is a means for accurately determining all types of secondary stresses in complicated structures that cannot be satisfactorily analyzed by simplified methods.

(2) It is sufficiently flexible to cope with a wide variety of structural problems involving nonuniform temperature distributions.

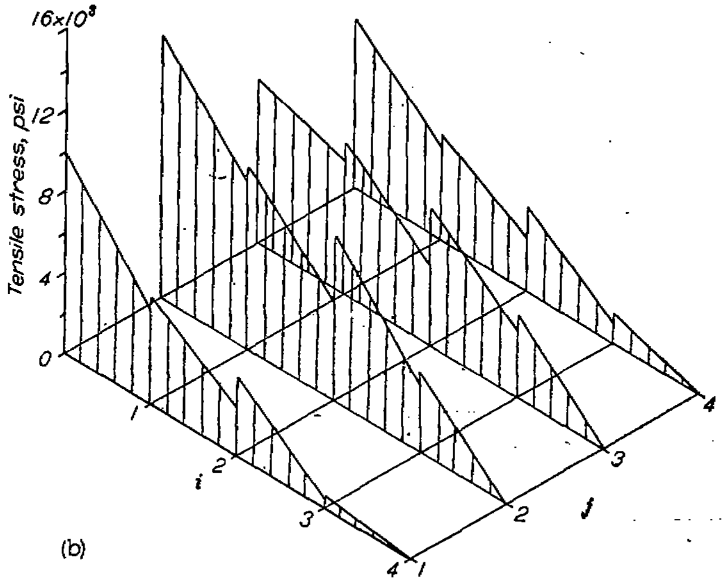
(3) It involves only simple arithmetic that can be handled by automatic computing machinery.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *September 12, 1950.*

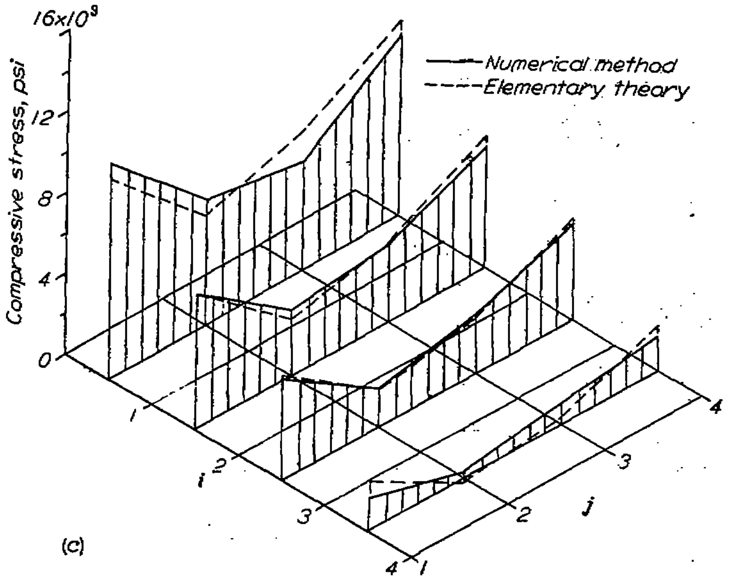


(a) Spanwise distribution of upper-surface stringer stress.

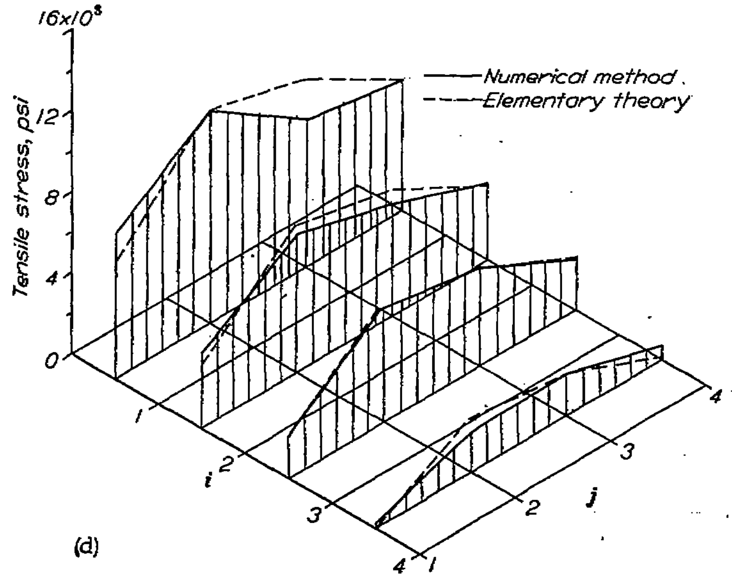
FIGURE 5.—Calculated stress distribution in the idealized structure.



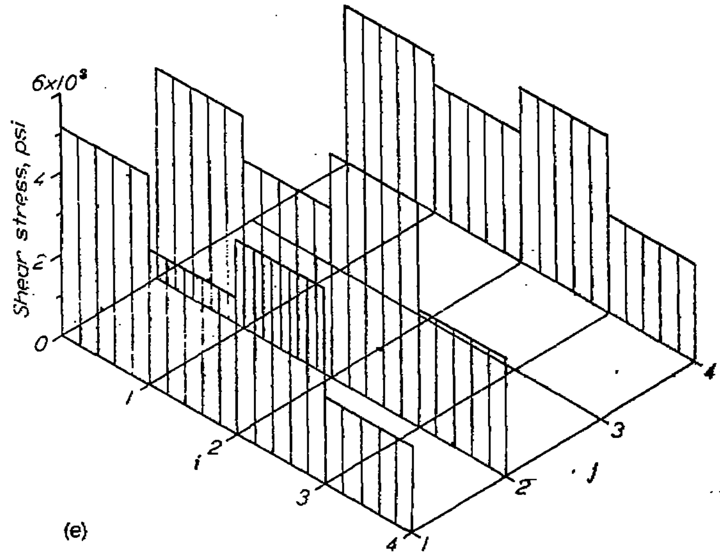
(b) Spanwise distribution of lower-surface stringer stress.
FIGURE 5.—Continued.



(c) Chordwise distribution of upper-surface normal stress.
FIGURE 5.—Continued.



(d) Chordwise distribution of lower-surface normal stress.
FIGURE 5.—Continued.



(e) Spanwise distribution of web shear stress.
FIGURE 5.—Concluded.

APPENDIX

DERIVATION OF GENERAL EQUATIONS

The general equations required for the numerical analysis of a stiffened shell of arbitrary cross section with any number of internal cells and under a nonuniform temperature distribution are developed. The basic assumptions and a general description of the method have been given previously and are not repeated.

SYMBOLS

A	cross-sectional area of stringer, square inches
b	width of panel on $\underline{k}$ grid line, inches
E	modulus of elasticity, psi
F	applied force, pounds
G	modulus of rigidity, psi
h	width of panel on $\underline{j}$ grid line, inches
I	moment of inertia, inches^4
J	shear stiffness parameters
l, m	coordinates of a special set of axes
L	length of bay, inches
M	applied moment, inch-pounds
P	axial load in stringer, positive for tensile load, pounds
Q	area moment, inches^3
r	normal distance to panel on $\underline{k}$ grid line, positive in positive z-direction, inches
T	temperature increment, measured from temperature of zero thermal stress which is 60° F in the example presented, degrees Fahrenheit
t	panel thickness, inches
u, v, w	displacements in x-, y-, and z-directions, respectively, inches
x, y, z	rectangular coordinate axes
α	coefficient of thermal expansion, inches per inch per degree Fahrenheit
β	angular rotation used in correction cycle, radians
γ	shear strain, radians
δ, Δ	increment

ϵ	normal strain, inches per inch
θ	angular rotation about x-axis, radians
λ	rotation of special set of axes, degrees or radians
ρ	normal distance to panel on $\underline{j}$ grid line, positive in positive y-direction, inches
σ	normal stress, positive for tensile stress, psi
τ	shear stress, positive in direction of associated coordinate axis when tensile stress on cross section is in positive x-direction, psi
ϕ	angle between normal line r and z-axis, degrees or radians
ψ	angle between normal line ρ and y-axis, degrees or radians

Subscripts:

i, j, k	grid system
x, y, z	coordinate axes
v, w, θ	bay displacements
0	initial value
n	cycle of iteration

A prime refers to the principal shear axes and two primes refer to the principal inertia axes. A bar over a symbol indicates an average value at the center of a bay.

NOTATION

The notation employed is illustrated in figure 6. The system adopted for designating parts of the structure is as follows:

Bulkheads divide the length of the structure into a number of bays. The subscript i is used to designate a given bulkhead or the bay between the $i-1$ and ith bulkheads.

The stringers and panels in a given cross section form the basis of a grid work which can be used to designate these elements. These grid lines are not necessarily straight, parallel, or perpendicular but follow the panels. Those grid lines that are approximately parallel to the z-axis are designated by the subscript j; those approximately parallel to the y-axis are designated by the subscript k.

With this system, points and stringers can be uniquely located as follows:

The point on the ith bulkhead at the intersection of the jth and kth grid lines is designated by the subscripts i,j,k.

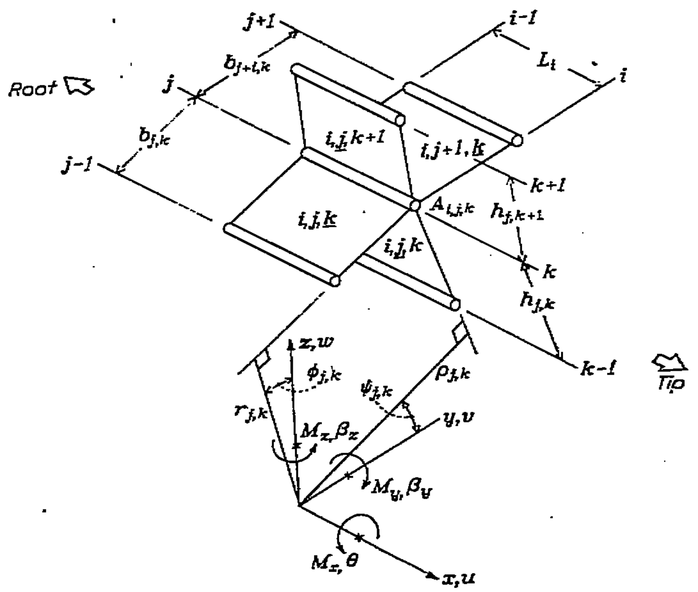


FIGURE 6.—Notation and coordinate system.

The stringer in the ith bay at the intersection of the jth and kth grid lines is designated by the subscripts i,j,k.

In order to locate a panel, the grid line on which it lies must be known. This notation consists of underlining the appropriate subscript; for example:

The panel in the ith bay on the jth grid line and between the $k-1$ and kth grid lines is designated by the subscripts $i,\underline{j},k$.

The panel in the ith bay on the kth grid line and between the $j-1$ and jth grid lines is designated by the subscripts $i,j,\underline{k}$.

The grid lines and bulkheads are numbered such that the numbers increase in the directions of the positive coordinate axes.

STRESS-STRAIN RELATIONSHIPS

The shear strain in a given panel is constant and is defined by the following relationships which depend upon the location of the panel:

$$\gamma_{i,j,\underline{k}}=\left(\frac{\tau}{G}\right)_{i,j,\underline{k}}=\frac{1}{2b_{j,k}}(u_{i,j,k}+u_{i-1,j,k}-u_{i,j-1,k}-u_{i-1,j-1,k})+\frac{\Delta v_i}{L_i}\cos\phi_{j,k}+\frac{\Delta w_i}{L_i}\sin\phi_{j,k}-\frac{\Delta\theta_i}{L_i}r_{j,k} \tag{A1a}$$

$$\gamma_{i,\underline{j},k}=\left(\frac{\tau}{G}\right)_{i,\underline{j},k}=\frac{1}{2h_{j,k}}(u_{i,j,k}+u_{i-1,j,k}-u_{i,j,k-1}-u_{i-1,j,k-1})-\frac{\Delta v_i}{L_i}\sin\psi_{j,k}+\frac{\Delta w_i}{L_i}\cos\psi_{j,k}+\frac{\Delta\theta_i}{L_i}\rho_{j,k} \tag{A1b}$$

When the shear strains are being computed, the normal distances r and ρ must be given their proper signs.

The constant shear stress produces a linearly varying strain in the stringer and its average value at the center of the bay is

$$\bar{\epsilon}_{i,j,k}=\frac{u_{i,j,k}-u_{i-1,j,k}}{L_i}=\left(\frac{\bar{P}}{AE}+\alpha T\right)_{i,j,k} \tag{A2}$$

Note that the thermal expansion is included in the relationship between stringer stress and strain.

EQUILIBRIUM OF INDIVIDUAL STRINGERS

If a half-bay length of stringer on each side of point (i, j, k) is isolated, the force system of figure 7 is obtained, and the following equilibrium equation can be written:

$$-\left(\frac{\tau tL}{2}\right)_{i,j,\underline{k}}+\left(\frac{\tau tL}{2}\right)_{i,j+1,\underline{k}}-\left(\frac{\tau tL}{2}\right)_{i,\underline{j},k}+\left(\frac{\tau tL}{2}\right)_{i,\underline{j},k+1}-\left(\frac{\tau tL}{2}\right)_{i+1,j,\underline{k}}+\left(\frac{\tau tL}{2}\right)_{i+1,j+1,\underline{k}}-\left(\frac{\tau tL}{2}\right)_{i+1,\underline{j},k}+\left(\frac{\tau tL}{2}\right)_{i+1,\underline{j},k+1}+\bar{P}_{i+1,j,k}-\bar{P}_{i,j,k}+(F_x)_{i,j,k}=0 \tag{A3}$$

Substituting equations (A1) and (A2) into equation (A3) yields the following equation for the stringer displacement of point (i, j, k) in terms of the displacements of the adjacent bays and stringers:

$$
\begin{aligned}
u_{i,j,k}=\frac{1}{\sum S_{i,j,k}}\Bigg\{ & u_{i-1,j-1,k}\left(\frac{GtL}{4b}\right)_{i,j,\underline{k}}+u_{i-1,j,k-1}\left(\frac{GtL}{4h}\right)_{i,j,k}+ \\
& u_{i-1,j,k}\left[\left(\frac{AE}{L}\right)_{i,j,k}-\left(\frac{GtL}{4b}\right)_{i,j,\underline{k}}-\left(\frac{GtL}{4b}\right)_{i,j+1,\underline{k}}-\left(\frac{GtL}{4h}\right)_{i,\underline{j},k}-\left(\frac{GtL}{4h}\right)_{i,\underline{j},k+1}\right]+u_{i-1,j,k+1}\left(\frac{GtL}{4h}\right)_{i,\underline{j},k+1}+ \\
& u_{i-1,j+1,k}\left(\frac{GtL}{4b}\right)_{i,j+1,\underline{k}}+u_{i,j-1,k}\left[\left(\frac{GtL}{4b}\right)_{i,j,\underline{k}}+\left(\frac{GtL}{4b}\right)_{i+1,j,\underline{k}}\right]+u_{i,j,k-1}\left[\left(\frac{GtL}{4h}\right)_{i,\underline{j},k}+\left(\frac{GtL}{4h}\right)_{i+1,\underline{j},k}\right]+ \\
& u_{i,j,k+1}\left[\left(\frac{GtL}{4h}\right)_{i,\underline{j},k+1}+\left(\frac{GtL}{4h}\right)_{i+1,\underline{j},k+1}\right]+u_{i,j+1,k}\left[\left(\frac{GtL}{4b}\right)_{i,j+1,\underline{k}}+\left(\frac{GtL}{4b}\right)_{i+1,j+1,\underline{k}}\right]+u_{i+1,j-1,k}\left(\frac{GtL}{4b}\right)_{i+1,j,\underline{k}}+ \\
& u_{i+1,j,k-1}\left(\frac{GtL}{4h}\right)_{i+1,\underline{j},k}+u_{i+1,j,k}\left[\left(\frac{AE}{L}\right)_{i+1,j,k}-\left(\frac{GtL}{4b}\right)_{i+1,j,\underline{k}}-\left(\frac{GtL}{4b}\right)_{i+1,j+1,\underline{k}}-\left(\frac{GtL}{4h}\right)_{i+1,\underline{j},k}-\left(\frac{GtL}{4h}\right)_{i+1,\underline{j},k+1}\right]+ \\
& u_{i+1,j,k+1}\left(\frac{GtL}{4h}\right)_{i+1,\underline{j},k+1}+u_{i+1,j+1,k}\left(\frac{GtL}{4b}\right)_{i+1,j+1,\underline{k}}+ \\
& \Delta v_i\left[\left(\frac{Gt\cos\phi}{2}\right)_{i,j+1,\underline{k}}-\left(\frac{Gt\cos\phi}{2}\right)_{i,j,\underline{k}}-\left(\frac{Gt\sin\psi}{2}\right)_{i,\underline{j},k+1}+\left(\frac{Gt\sin\psi}{2}\right)_{i,\underline{j},k}\right]+ \\
& \Delta v_{i+1}\left[\left(\frac{Gt\cos\phi}{2}\right)_{i+1,j+1,\underline{k}}-\left(\frac{Gt\cos\phi}{2}\right)_{i+1,j,\underline{k}}-\left(\frac{Gt\sin\psi}{2}\right)_{i+1,\underline{j},k+1}+\left(\frac{Gt\sin\psi}{2}\right)_{i+1,\underline{j},k}\right]+ \\
& \Delta w_i\left[\left(\frac{Gt\sin\phi}{2}\right)_{i,j+1,\underline{k}}-\left(\frac{Gt\sin\phi}{2}\right)_{i,j,\underline{k}}+\left(\frac{Gt\cos\psi}{2}\right)_{i,\underline{j},k+1}-\left(\frac{Gt\cos\psi}{2}\right)_{i,\underline{j},k}\right]+ \\
& \Delta w_{i+1}\left[\left(\frac{Gt\sin\phi}{2}\right)_{i+1,j+1,\underline{k}}-\left(\frac{Gt\sin\phi}{2}\right)_{i+1,j,\underline{k}}+\left(\frac{Gt\cos\psi}{2}\right)_{i+1,\underline{j},k+1}-\left(\frac{Gt\cos\psi}{2}\right)_{i+1,\underline{j},k}\right]- \\
& \Delta\theta_i\left[\left(\frac{Gtr}{2}\right)_{i,j+1,\underline{k}}-\left(\frac{Gtr}{2}\right)_{i,j,\underline{k}}-\left(\frac{Gt\rho}{2}\right)_{i,\underline{j},k+1}+\left(\frac{Gt\rho}{2}\right)_{i,\underline{j},k}\right]- \\
& \Delta\theta_{i+1}\left[\left(\frac{Gtr}{2}\right)_{i+1,j+1,\underline{k}}-\left(\frac{Gtr}{2}\right)_{i+1,j,\underline{k}}-\left(\frac{Gt\rho}{2}\right)_{i+1,\underline{j},k+1}+\left(\frac{Gt\rho}{2}\right)_{i+1,\underline{j},k}\right]+(AE\alpha T)_{i,j,k}-(AE\alpha T)_{i+1,j,k}+(F_x)_{i,j,k}\Bigg\} \quad \text{(A4)}
\end{aligned}
$$

where

$$
\begin{aligned}
\sum S_{i,j,k}= & \left(\frac{AE}{L}\right)_{i,j,k}+\left(\frac{GtL}{4b}\right)_{i,j,\underline{k}}+\left(\frac{GtL}{4b}\right)_{i,j+1,\underline{k}}+\left(\frac{GtL}{4h}\right)_{i,\underline{j},k}+\left(\frac{GtL}{4h}\right)_{i,\underline{j},k+1}+ \\
& \left(\frac{AE}{L}\right)_{i+1,j,k}+\left(\frac{GtL}{4b}\right)_{i+1,j,\underline{k}}+\left(\frac{GtL}{4b}\right)_{i+1,j+1,\underline{k}}+\left(\frac{GtL}{4h}\right)_{i+1,\underline{j},k}+\left(\frac{GtL}{4h}\right)_{i+1,\underline{j},k+1}
\end{aligned}
$$

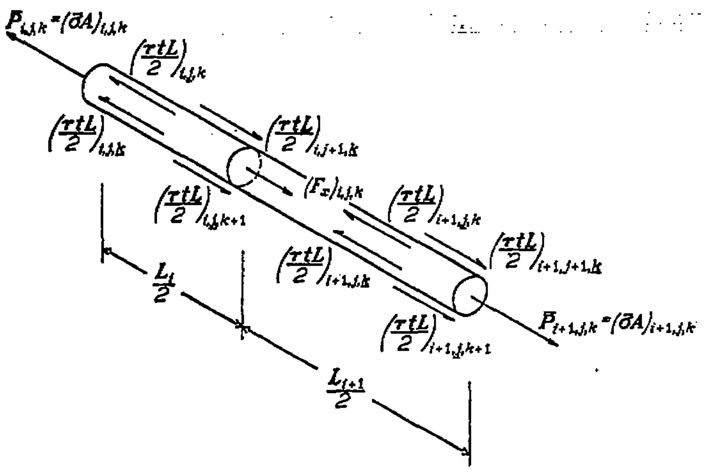


FIGURE 7.—Forces on stringer.

Equation (A4) involves no assumptions regarding equality of structural dimensions, temperatures, or elastic properties about point (i, j, k). If any element is missing, the associated stiffness goes to zero and the general equation is still applicable. Since AE and Gt always appear as products, the variation of elastic properties with temperature is equivalent to changes in the stringer areas and sheet thicknesses of the effective structure. Furthermore, the thermal-expansion terms appear in the same manner as axial loads applied to the stringers. Thus, if desired, the effects of a nonuniform temperature distribution can be determined by applying a set of equivalent loads to a new effective structure.

BAY SHEAR AND TORQUE EQUILIBRIUM

The equations for the bay displacements (v,w,θ) can be obtained from equilibrium of the shear forces on the bay cross section

$$(F_y)_i-\sum_j\sum_k[(\tau tb\cos\phi)_{i,j,\underline{k}}-(\tau th\sin\psi)_{i,\underline{j},k}]=0 \quad \text{(A5a)}$$

$$(F_z)_i-\sum_j\sum_k[(\tau tb\sin\phi)_{i,j,\underline{k}}+(\tau th\cos\psi)_{i,\underline{j},k}]=0 \quad \text{(A5b)}$$

$$(M_x)_i+\sum_j\sum_k[(\tau tbr)_{i,j,\underline{k}}-(\tau th\rho)_{i,\underline{j},k}]=0 \quad \text{(A5c)}$$

Substitution of equations (A1) for the shear stresses in equations (A5) results in

$$(J_{vv})_i\Delta v_i+(J_{vw})_i\Delta w_i-(J_{\theta v})_i\Delta\theta_i=(F_y)_i+\sum_j\sum_k(u_{i,j,k}+u_{i-1,j,k})\left[\left(\frac{Gt\cos\phi}{2}\right)_{i,j+1,\underline{k}}-\left(\frac{Gt\cos\phi}{2}\right)_{i,j,\underline{k}}-\left(\frac{Gt\sin\psi}{2}\right)_{i,\underline{j},k+1}+\left(\frac{Gt\sin\psi}{2}\right)_{i,\underline{j},k}\right] \quad \text{(A6a)}$$

$$(J_{vw})_i\Delta v_i+(J_{ww})_i\Delta w_i-(J_{\theta w})_i\Delta\theta_i=(F_z)_i+\sum_j\sum_k(u_{i,j,k}+u_{i-1,j,k})\left[\left(\frac{Gt\sin\phi}{2}\right)_{i,j+1,\underline{k}}-\left(\frac{Gt\sin\phi}{2}\right)_{i,j,\underline{k}}+\left(\frac{Gt\cos\psi}{2}\right)_{i,\underline{j},k+1}-\left(\frac{Gt\cos\psi}{2}\right)_{i,\underline{j},k}\right] \quad \text{(A6b)}$$

$$-(J_{\theta v})_i\Delta v_i-(J_{\theta w})_i\Delta w_i+(J_{\theta\theta})_i\Delta\theta_i=(M_x)_i-\sum_j\sum_k(u_{i,j,k}+u_{i-1,j,k})\left[\left(\frac{Gtr}{2}\right)_{i,j+1,\underline{k}}-\left(\frac{Gtr}{2}\right)_{i,j,\underline{k}}-\left(\frac{Gt\rho}{2}\right)_{i,\underline{j},k+1}+\left(\frac{Gt\rho}{2}\right)_{i,\underline{j},k}\right] \quad \text{(A6c)}$$

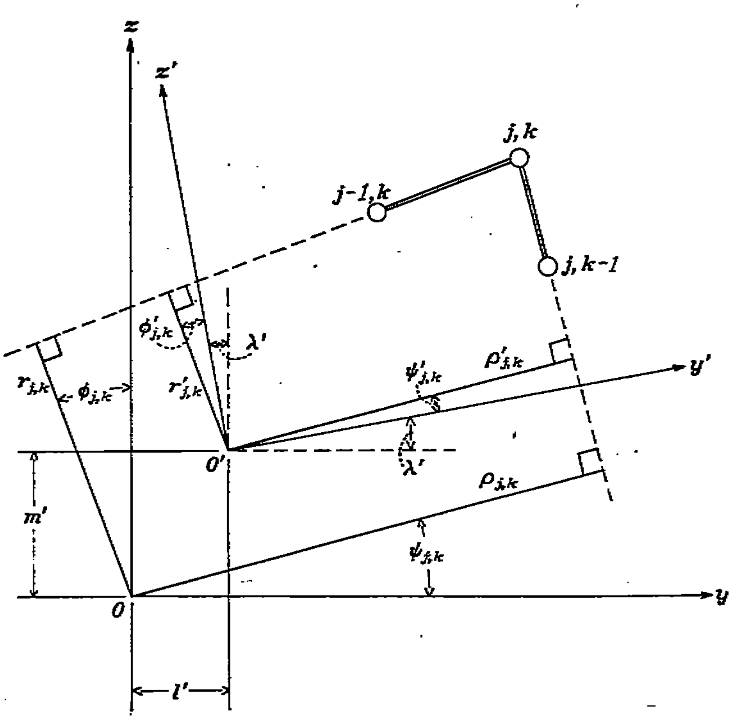


FIGURE 8.—Notation used to locate principal shear axes.

where

$$(J_{vv})_i=\sum\sum_k\left[\left(\frac{Gtb\cos^2\phi}{L}\right)_{i,j,\underline{k}}+\left(\frac{Gth\sin^2\psi}{L}\right)_{i,\underline{j},k}\right]$$

$$(J_{vw})_i=\sum_j\sum_k\left[\left(\frac{Gtb\sin\phi\cos\phi}{L}\right)_{i,j,\underline{k}}-\left(\frac{Gth\sin\psi\cos\psi}{L}\right)_{i,\underline{j},k}\right]$$

$$(J_{ww})_i=\sum_j\sum_k\left[\left(\frac{Gtb\sin^2\phi}{L}\right)_{i,j,\underline{k}}+\left(\frac{Gth\cos^2\psi}{L}\right)_{i,\underline{j},k}\right]$$

$$(J_{\theta v})_i=\sum_j\sum_k\left[\left(\frac{Gtbr\cos\phi}{L}\right)_{i,j,\underline{k}}+\left(\frac{Gth\rho\sin\psi}{L}\right)_{i,\underline{j},k}\right]$$

$$(J_{\theta w})_i=\sum_j\sum_k\left[\left(\frac{Gtbr\sin\phi}{L}\right)_{i,j,\underline{k}}-\left(\frac{Gth\rho\cos\psi}{L}\right)_{i,\underline{j},k}\right]$$

$$(J_{\theta\theta})_i=\sum_j\sum_k\left[\left(\frac{Gtbr^2}{L}\right)_{i,j,\underline{k}}+\left(\frac{Gth\rho^2}{L}\right)_{i,\underline{j},k}\right]$$

Equations (A6) can be simplified by eliminating the coupling terms if the axes used in the computations are the principal shear axes of the cross section. These axes are defined such that

$$J_{vw}'=J_{\theta v}'=J_{\theta w}'=0 \quad \text{(A7)}$$

The relationship between the location and orientation of points and panels in two systems of coordinates, arbitrary axes (x,y,z) and the principal shear axes (x',y',z'), is shown in figure 8 and given by the following equations:

$$y'=(z-m')\sin\lambda'+(y-l')\cos\lambda' \quad \text{(A8a)}$$

$$z'=(z-m')\cos\lambda'-(y-l')\sin\lambda' \quad \text{(A8b)}$$

$$\phi'=\phi-\lambda' \quad \text{(A9a)}$$

$$\psi'=\psi-\lambda' \quad \text{(A9b)}$$

$$r'=r+l'\sin\phi-m'\cos\phi \quad \text{(A10a)}$$

$$\rho'=\rho-l'\cos\psi-m'\sin\psi \quad \text{(A10b)}$$

Then the location of the principal shear axes is

$$\tan 2\lambda'=\frac{2J_{vw}}{J_{vv}-J_{ww}} \quad \text{(A11a)}$$

$$l'=-\frac{J_{vv}J_{\theta w}-J_{vw}J_{\theta v}}{J_{vv}J_{ww}-J_{vw}^2} \quad \text{(A11b)}$$

$$m'=\frac{J_{ww}J_{\theta v}-J_{vw}J_{\theta w}}{J_{vv}J_{ww}-J_{vw}^2} \quad \text{(A11c)}$$

and, with respect to these axes,

$$J_{vv}'=J_{vv}\cos^2\lambda'+J_{ww}\sin^2\lambda'+2J_{vw}\sin\lambda'\cos\lambda' \quad \text{(A12a)}$$

$$J_{ww}'=J_{vv}\sin^2\lambda'+J_{ww}\cos^2\lambda'-2J_{vw}\sin\lambda'\cos\lambda' \quad \text{(A12b)}$$

$$J_{\theta\theta}'=J_{\theta\theta}+l'J_{\theta w}-m'J_{\theta v} \quad \text{(A12c)}$$

$$F_y'=F_z\sin\lambda'+F_y\cos\lambda' \quad \text{(A13a)}$$

$$F_z'=F_z\cos\lambda'-F_y\sin\lambda' \qquad \text{(A13b)}$$

$$M_x'=M_x+m'F_y-l'F_z \qquad \text{(A13c)}$$

When referred to the principal shear axes, the equations for the bay displacements become

$$\Delta v_i'=\left(\frac{1}{J_{vv}'}\right)_i\left\{(F_y')_i+\sum_j\sum_k(u_{i,j,k}+u_{i-1,j,k})\left[\left(\frac{Gt\cos\phi'}{2}\right)_{i,j+1,\underline{k}}-\left(\frac{Gt\cos\phi'}{2}\right)_{i,j,\underline{k}}-\left(\frac{Gt\sin\psi'}{2}\right)_{i,\underline{j},k+1}+\left(\frac{Gt\sin\psi'}{2}\right)_{i,\underline{j},k}\right]\right\} \qquad \text{(A14a)}$$

$$\Delta w_i'=\left(\frac{1}{J_{ww}'}\right)_i\left\{(F_z')_i+\sum_j\sum_k(u_{i,j,k}+u_{i-1,j,k})\left[\left(\frac{Gt\sin\phi'}{2}\right)_{i,j+1,\underline{k}}-\left(\frac{Gt\sin\phi'}{2}\right)_{i,j,\underline{k}}+\left(\frac{Gt\cos\psi'}{2}\right)_{i,\underline{j},k+1}-\left(\frac{Gt\cos\psi'}{2}\right)_{i,\underline{j},k}\right]\right\} \qquad \text{(A14b)}$$

$$\Delta\theta_i'=\left(\frac{1}{J_{\theta\theta}'}\right)_i\left\{(M_x')_i-\sum_j\sum_k(u_{i,j,k}+u_{i-1,j,k})\left[\left(\frac{Gtr'}{2}\right)_{i,j+1,\underline{k}}-\left(\frac{Gtr'}{2}\right)_{i,j,\underline{k}}-\left(\frac{Gt\rho'}{2}\right)_{i,\underline{j},k+1}+\left(\frac{Gt\rho'}{2}\right)_{i,\underline{j},k}\right]\right\} \qquad \text{(A14c)}$$

BAY THRUST AND MOMENT EQUILIBRIUM

The equations obtained from equations (A4) and (A14) are sufficient in themselves to define completely the displacements of the structure. However, if the equations are solved by iteration, it is helpful to employ a periodic correction cycle based on the gross equilibrium of axial loads in the cross section

$$(F_x)_i-\sum_j\sum_k(\overline{P})_{i,j,k}=0 \qquad \text{(A15a)}$$

$$(\overline{M}_y)_i-\sum_j\sum_k(\overline{P}z)_{i,j,k}=0 \qquad \text{(A15b)}$$

$$(\overline{M}_z)_i+\sum_j\sum_k(\overline{P}y)_{i,j,k}=0 \qquad \text{(A15c)}$$

It can be shown that equations (A15) are satisfied by the solution of equations (A4) and (A14); however, they are not likely to be satisfied by the displacement values obtained from any given cycle of iteration. In reference 2 it was demonstrated that convergence of the iterative process can be speeded if the displacement values are periodically corrected so that the stringer displacements satisfy equations (A15).

The corrections applied to the stringer displacements are a planar distribution over the cross section and are determined as follows:

$$(u_{i,j,k})_{n+1}=(u_{i,j,k})_n+\Delta u_{i,j,k} \qquad \text{(A16)}$$

where

$$\Delta u_{i,j,k}=\delta u_i+\beta_{z,i}\,y_{j,k}+\beta_{y,i}\,z_{j,k}$$

Substituting equation (A16) into (A15) yields

$$(AE)_i\,\delta u_i+(EQ_z)_i\beta_{z,i}+(EQ_y)_i\beta_{y,i}=(LF_x)_i+L_i\sum_j\sum_k\left[(AE\alpha T)_{i,j,k}-(u_{i,j,k}-u_{i-1,j,k})\left(\frac{AE}{L}\right)_{i,j,k}\right] \qquad \text{(A17a)}$$

$$(EQ_y)_i\,\delta u_i+(EI_{yz})_i\beta_{z,i}+(EI_{yy})_i\beta_{y,i}=(L\overline{M}_y)_i+L_i\sum_j\sum_k\left[(AE\alpha Tz)_{i,j,k}-(u_{i,j,k}-u_{i-1,j,k})\left(\frac{AEz}{L}\right)_{i,j,k}\right] \qquad \text{(A17b)}$$

$$(EQ_z)\delta u_i+(EI_{zz})_i\beta_{z,i}+(EI_{yz})_i\beta_{y,i}=-(L\overline{M}_z)_i+L_i\sum_j\sum_k\left[(AE\alpha Ty)_{i,j,k}-(u_{i,j,k}-u_{i-1,j,k})\left(\frac{AEy}{L}\right)_{i,j,k}\right] \qquad \text{(A17c)}$$

where

$$(AE)_i=\sum_j\sum_k(AE)_{i,j,k} \qquad (EI_{yy})_i=\sum_j\sum_k(AEz^2)_{i,j,k}$$

$$(EQ_z)_i=\sum_j\sum_k(AEy)_{i,j,k} \qquad (EI_{zz})_i=\sum_j\sum_k(AEy^2)_{i,j,k}$$

$$(EQ_y)_i=\sum_j\sum_k(AEz)_{i,j,k} \qquad (EI_{yz})_i=\sum_j\sum_k(AEyz)_{i,j,k}$$

These equations can be simplified by elimination of the coupling terms if the computations are referred to the equivalent principal inertia axes of the cross section. These axes are referred to as equivalent because the variation of modulus of elasticity over the cross section is taken into consideration. These axes (x'',y'',z'') are defined such that

$$EQ_z''=EQ_y''=EI_{yz}''=0 \qquad \text{(A18)}$$

and then the following relationships are applicable:

$$y''=(z-m'')\sin\lambda''+(y-l'')\cos\lambda'' \qquad \text{(A19a)}$$

$$z''=(z-m'')\cos\lambda''-(y-l'')\sin\lambda'' \qquad \text{(A19b)}$$

$$\tan 2\lambda''=\frac{2(EI_{yz}-AEl''m'')}{(EI_{yy}-AEm''^2)-(EI_{zz}-AEl''^2)} \qquad \text{(A20a)}$$

$$l''=\frac{EQ_z}{AE} \qquad \text{(A20b)}$$

$$m''=\frac{EQ_y}{AE} \qquad \text{(A20c)}$$

$$EI_{yy}''=(EI_{yy}-AEm''^2)\cos^2\lambda''+(EI_{zz}-AEl''^2)\sin^2\lambda''-2(EI_{yz}-AEl''m'')\sin\lambda''\cos\lambda'' \qquad \text{(A21a)}$$

$$EI_{zz}''=(EI_{yy}-AEm''^2)\sin^2\lambda''+(EI_{zz}-AEl''^2)\cos^2\lambda''+2(EI_{yz}-AEl''m'')\sin\lambda''\cos\lambda'' \qquad \text{(A21b)}$$

$$F_x''=F_x \qquad \text{(A22a)}$$

$$\overline{M}_y''=\overline{M}_z\sin\lambda''+\overline{M}_y\cos\lambda'' \qquad \text{(A22b)}$$

$$\overline{M}_z''=\overline{M}_z\cos\lambda''-\overline{M}_y\sin\lambda'' \qquad \text{(A22c)}$$

A further simplification of the correction cycle can be made by eliminating the load and temperature terms on the right-hand side of equations (A17). This elimination can be accomplished by iterating the difference between the exact solution and one which satisfies statics (equations (A15)) but not necessarily continuity. The iteration of differences has an additional advantage in that smaller numbers, and consequently less work, are required to obtain a solution.

An examination of equations (A17) indicates that they can be satisfied by a planar distribution of strain corresponding to the elementary analysis of reference 1. Then the initial values of stringer displacements u can be defined as follows:

$$u_{i,j,k}=u_{i-1,j,k}+(\epsilon_0 L)_{i,j,k} \tag{A23}$$

where

$$(\epsilon_0 L)_{i,j,k}=(\delta u_i'')_0+(\beta_{z,i}'')_0 y_{j,k}''+(\beta_{y,i}'')_0 z_{j,k}''$$

and, with respect to the equivalent principal inertia axes,

$$(\delta u_i'')_0=\left(\frac{L}{AE}\right)_i\left[(F_x'')_i+\sum_j\sum_k(AE\alpha T)_{i,j,k}\right] \tag{A24a}$$

$$(\beta_{z,i}'')_0=\left(\frac{L}{EI_{zz}''}\right)_i\left[-(\overline{M}_z'')_i+\sum_j\sum_k(AE\alpha Ty'')_{i,j,k}\right] \tag{A24b}$$

$$(\beta_{y,i}'')_0=\left(\frac{L}{EI_{yy}''}\right)_i\left[(\overline{M}_y'')_i+\sum_j\sum_k(AE\alpha Tz'')_{i,j,k}\right] \tag{A24c}$$

The corresponding values of the bay displacements are obtained from equations (A14) and (A23).

Then the correction-cycle equations applicable to the iterated differences are as follows:

$$\delta u_i''=-\left(\frac{1}{AE}\right)_i\sum_j\sum_k(\Delta u_{i,j,k}-\Delta u_{i-1,j,k})(AE)_{i,j,k} \tag{A25a}$$

$$\beta_{z,i}''=-\left(\frac{1}{EI_{zz}''}\right)_i\sum_j\sum_k(\Delta u_{i,j,k}-\Delta u_{i-1,j,k})(AEy'')_{i,j,k} \tag{A25b}$$

$$\beta_{y,i}''=-\left(\frac{1}{EI_{yy}''}\right)_i\sum_j\sum_k(\Delta u_{i,j,k}-\Delta u_{i-1,j,k})(AEz'')_{i,j,k} \tag{A25c}$$

Equations (A25) provide corrections to the stringer displacements u only. These corrections remove any unbalanced moment or thrust on the cross section but add unbalanced shear forces which are removed by correcting the bay displacements (v,w,θ). The corrected bay displacements are obtained from the corrected stringer displacements by application of equation (A14). These two operations constitute the complete correction cycle that brings the stringer loads into equilibrium with the external loads without changing the shear stress in any panel.

REFERENCES

1. Heldenfels, Richard R.: The Effect of Nonuniform Temperature Distributions on the Stresses and Distortions of Stiffened-Shell Structures. NACA TN 2240, 1950.
2. Duberg, John E.: A Numerical Procedure for the Stress Analysis of Stiffened Shells. Jour. Aero. Sci., vol. 16, no. 8, Aug. 1949, pp. 451–462.
3. Southwell, R. V.: Relaxation Methods in Engineering Science. A Treatise on Approximate Computations. The Clarendon Press (Oxford), 1940.

TABLE I.—STRINGER PROPERTIES

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
i	L	j	T	E	αT	A	AE	$\frac{AE}{L}$	$AE\alpha T$
				*	**		(5)×(7)	$\frac{(8)}{(2)}$	(6)×(8)
1	20	1	206	9.55640×10⁶	2,728.5×10⁻⁶	1.11	10.60760×10⁶	530,380	23,943
	20	2	158	9.89078	2,066.0	1.15	11.37440	568,720	23,500
	20	3	155	9.90937	2,025.2	.74	7.33293	366,647	14,851
	20	4	162	9.86553	2,120.6	1.01	9.96424	498,212	21,130
2	20	1	219	9.45386	2,910.7	0.98	9.26478	463,239	26,967
	20	2	173	9.79376	2,271.3	1.05	10.28345	514,173	23,357
	20	3	165	9.84635	2,161.6	.66	6.49859	324,930	14,047
	20	4	167	9.83339	2,189.0	.89	8.75172	437,586	19,158
3	20	1	231	9.35168	3,080.0	0.59	5.51926	275,963	16,999
	20	2	187	9.69708	2,464.3	.62	6.01219	300,610	14,816
	20	3	175	9.78031	2,298.8	.38	3.71652	185,826	8,544
	20	4	172	9.80044	2,257.6	.54	5.29224	264,612	11,948
4	20	1	244	9.24233	3,264.4	0.46	4.25147	212,574	13,878
	20	2	202	9.58601	2,672.7	.52	4.98520	249,260	13,324
	20	3	185	9.71125	2,436.7	.30	2.91338	145,669	7,099
	20	4	177	9.76674	2,826.3	.42	4.10203	205,102	9,543

*$E=(10.5-0.00147T-0.0000151T^2)\times10^6$
**$\alpha T=(12.52T+0.00352T^2)\times10^{-6}$

TABLE II.—PANEL PROPERTIES

			Cover: panels$_{i,j,1}$								Webs: panels$_{i,j,1}$							
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)
i	L	j	T	G	t	b	$\frac{Gt}{2}$	$\frac{Gtb}{L}$	$\frac{GtL}{4b}$	$\frac{2}{tL}$	T	G	t	h	$\frac{Gt}{2}$	$\frac{Gth}{L}$	$\frac{GtL}{4h}$	$\frac{2}{tL}$
				*			$\frac{(15)\times(16)}{2}$	$\frac{2(17)\times(18)}{(12)}$	$\frac{(18)\times(12)}{2(17)}$	$\frac{2}{(16)\times(12)}$		*			$\frac{(23)\times(24)}{2}$	$\frac{2(25)\times(26)}{(12)}$	$\frac{(26)\times(12)}{2(25)}$	$\frac{2}{(24)\times(12)}$
1	20	1									206	3.49967×10⁶	0.064	10	111,989	111,989	111,989	1.56250
	20	2	182	3.57892×10⁶	0.040	10	71,578	71,578	71,578	2.5	158	3.65265	.051	10	93,143	93,143	93,143	1.96078
	20	3	157	3.65560	.040	10	73,112	73,112	73,112	2.5								
	20	4	159	3.64969	.040	10	72,994	72,994	72,994	2.5	162	3.64075	.051	10	92,839	92,839	92,839	1.96078
2	20	1									219	3.45443	0.064	10	110,542	110,542	110,542	1.56250
	20	2	196	3.53336	0.040	10	70,667	70,667	70,667	2.5	173	3.60722	.051	10	91,984	91,984	91,984	1.96078
	20	3	169	3.61955	.040	10	72,391	72,391	72,391	2.5								
	20	4	166	3.62869	.040	10	72,574	72,574	72,574	2.5	167	3.62565	.051	10	92,454	92,454	92,454	1.96078
3	20	1									231	3.41122	0.032	10	54,580	54,580	54,580	3.12500
	20	2	209	3.45937	0.020	10	34,594	34,594	34,594	5.0	187	3.56267	.025	10	44,536	44,536	44,536	4.00000
	20	3	181	3.58211	.020	10	35,821	35,821	35,821	5.0								
	20	4	173	3.60722	.020	10	36,072	36,072	36,072	5.0	172	3.61032	.025	10	45,129	45,129	45,129	4.00000
4	20	1									244	3.36287	0.032	10	53,806	53,806	53,806	3.12500
	20	2	223	3.44018	0.020	10	34,402	34,402	34,402	5.0	202	3.51326	.025	10	43,916	43,916	43,916	4.00000
	20	3	194	3.53999	.020	10	35,400	35,400	35,400	5.0								
	20	4	181	3.58211	.020	10	35,821	35,821	35,821	5.0	177	3.59474	.025	10	44,934	44,934	44,934	4.00000

*$G=(4.0-0.00144T-0.0000048T^2)\times10^6$

TABLE III.—PRINCIPAL SHEAR AXES AND BAY DISPLACEMENT EQUATIONS

		Load problem*																			Temperature problem**	
(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)	(44)	(45)	(46)	(47)	(48)	(49)	(50)	(51)	(52)
i	j	$J_{ww}=J_{ww}'$	y	z, z', z''	$J_{\theta w}$	l'	y'	$J_{\theta\theta}'$	$F_z=F_z'$	M_z	M_z'	$\left(\frac{Gt}{2}\right)_{j+1,1}-\left(\frac{Gt}{2}\right)_{j,1}$	$\left(\frac{Gtz'}{2}\right)_{j,1}$	$\left(\frac{Gtz'}{2}\right)_{j+1,1}-\left(\frac{Gtz'}{2}\right)_{j,1}$	$\left(\frac{Gty'}{2}\right)_{j,1}$		$\Delta w'$ $[C]$	$\Delta w'$ $\{c\}$	$\Delta\theta'$ $[C]$	$\Delta\theta'$ $\{c\}$	$J_{vv}=J_{vv}'$	$\Delta v'$ $[C]$
		$\sum_{j=1}^{4}(27)$			$-\sum_{j=1}^{4}(27)(33)$	$-\frac{(35)}{(32)}$	$(33)-(36)$	$50\sum_{j=1}^{4}(28)+\sum_{j=1}^{4}(27)\times(37)^2$			$(40)-(36)\times(39)$	$(18)_{j+1}-(18)_j$	$5(18)$	$5(42)$	$(18)\times(37)$	$(45)+(44)$	$-\frac{2(43)}{(32)}$	$\frac{(39)}{(32)}$	$-\frac{2(46)}{(38)}$	$\frac{(41)}{(38)}$	$2\sum_{j=1}^{4}(19)$	$\frac{2(42)}{(51)}$
1	1	297, 971	−15	5	752, 965	−2. 526974	−12. 473026	57, 396, 352	8, 000	−40, 000	−19, 784	71, 578		357, 890	−1, 396, 841	−1, 038, 951	−0. 751678	0. 026848	0. 036202	−0. 000345	435, 368	0. 328816
	2		−5	5			−2. 473026					1, 534	357, 890	7, 670	−230, 345	−222, 675	−. 625182		. 007760			. 007047
	3		5	5			7. 526974					−118	365, 560	−590		−590			. 000020			−. 000542
	4		15	5			17. 526974					−72, 994	364, 970	−364, 970	1, 627, 187	1, 262, 217	−. 623142		−. 043982			−. 335321
2	1	294, 980	−15	5	731, 240	−2. 478948	−12. 521052	56, 942, 594	6, 000	−30, 000	−15, 126	70, 667		353, 335	−1, 384, 102	−1, 030, 767	−0. 749488	0. 020340	0. 036204	−0. 000266	431, 264	0. 327720
	2		−5	5			−2. 521052					1, 724	353, 335	8, 620	−231, 896	−223, 276	−. 623662		. 007842			. 007995
	3		5	5			7. 478948					183	361, 955	915		915			−. 000032			. 000849
	4		15	5			17. 478948					−72, 574	362, 870	−362, 870	1, 615, 999	1, 253, 129	−. 626850		−. 044014			−. 336564
3	1	144, 246	−15	5	364, 445	−2. 526569	−12. 473431	27, 966, 479	4, 000	−20, 000	−9, 894	34, 894		174, 470	−680, 800	−506, 330	−0. 756768	0. 027731	0. 036210	−0. 000354	213, 574	0. 326763
	2		−5	5			−2. 473431					927	174, 470	4, 635	−110, 157	−105, 522	−. 617504		. 007546			. 008681
	3		5	5			7. 526569					251	179, 105	1, 255		1, 255			−. 000090			. 002350
	4		15	5			17. 526569					−36, 072	180, 360	−180, 360	790, 957	610, 597	−. 625728		−. 043666			−. 337794
4	1	142, 656	−15	5	352, 660	−2. 472101	−12. 527899	27, 723, 739	2, 000	−10, 000	−5, 056	34, 402		172, 010	−674, 076	−502, 066	−0. 754346	0. 014020	0. 036220	−0. 000182	211, 246	0. 325706
	2		−5	5			−2. 527899					998	172, 010	4, 990	−111, 015	−106, 025	−. 615690		. 007648			. 009449
	3		5	5			7. 472101					421	177, 000	2, 105		2, 105			−. 000142			. 003986
	4		15	5			17. 472101					−35, 821	179, 105	−179, 105	785, 091	605, 986	−. 629962		−. 043716			−. 339140

*For load problem:

$$J_{ww}=\sum_{j=1}^{4}\left(\frac{Gth}{L}\right)_{j,1}$$

$$J_{\theta w}=-\sum_{j=1}^{4}\left(\frac{Gthy}{L}\right)_{j,1}$$

$$J_{\theta\theta}=\sum_{j=1}^{4}\left[2\left(\frac{Gtbz^2}{L}\right)_{j,1}+\left(\frac{Gthy^2}{L}\right)_{j,1}\right]$$

$$\Delta w_i'=\left(\frac{1}{J_{ww}'}\right)_i\left[(F_z')_i-2\sum_{j=1}^{4}(u_{i,j,1}+u_{i-1,j,1})\left(\frac{Gt}{2}\right)_{i,j,1}\right]$$

$$\Delta\theta_i'=\left(\frac{1}{J_{\theta\theta}'}\right)_i\left\{(M_z')_i-2\sum_{j=1}^{4}(u_{i,j,1}+u_{i-1,j,1})\left[\left(\frac{Gtz'}{2}\right)_{i,j+1,1}-\left(\frac{Gtz'}{2}\right)_{i,j,1}+\left(\frac{Gty'}{2}\right)_{i,j,1}\right]\right\}$$

**For temperature problem.

$$J_{vv}=2\sum_{j=1}^{4}\left(\frac{Gtb}{L}\right)_{j,1}$$

$$\Delta v_i'=\left(\frac{1}{J_{vv}'}\right)_i 2\sum_{j=1}^{4}(u_{i,j,1}+u_{i-1,j,1})\left[\left(\frac{Gt}{2}\right)_{i,j+1,1}-\left(\frac{Gt}{2}\right)_{i,j,1}\right]$$

Checks:

$$J_{\theta w}'=-\sum_{j=1}^{4}(27)\times(37)=0$$

$$J_{\theta\theta}'=J_{\theta\theta}+l'J_{\theta w}$$

$$\sum_{j=1}^{4}(42)=\sum_{j=1}^{4}(44)=\sum_{j=1}^{4}(46)=0$$

$$\sum_{j=1}^{4}(47)=\sum_{j=1}^{4}(49)=\sum_{j=1}^{4}(52)=0$$

TABLE IV.—PRINCIPAL INERTIA AXES, CORRECTION CYCLE, AND INITIAL STRINGER DISPLACEMENTS

		Load problem*						Temperature problem**											
53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
i	j	AEz''	$(EI_{yy}'')_i$	$\beta_{y,i}$	$\bar{M}_y=\bar{M}_y''$	$(\beta_{y,i})_0$	$(u_{i,j,1})_0$	$(AE)_i$	l''	y''	AEy''	$(EI_{zz}'')_i$	δu_i	$\beta_{z,i}$	$\sum_{j=1}^{4} AE\alpha T$	$\sum_{j=1}^{4} AE\alpha Ty''$	$(\delta u_i)_0$	$(\beta_{z,i})_0$	$(u_{i,j,1})_0$
		5 [illegible]	$50\sum_{j=1}^{4}$ [illegible]	$-\frac{2 [illegible]}{[illegible]}$		$\frac{[illegible]}{[illegible]}$	[illegible]$_{i-1}$ + 5 [illegible]	$2\sum_{j=1}^{4}$ [illegible]	$\frac{2\sum_{j=1}^{4} [illegible]}{[illegible]}$	[illegible] − [illegible]	[illegible] × [illegible]	$2\sum_{j=1}^{4}$ [illegible] × [illegible]	$-\frac{2 [illegible]}{[illegible]}$	$-\frac{2 [illegible]}{[illegible]}$	$\sum_{j=1}^{4}$ [illegible]	$\sum_{j=1}^{4}$ [illegible] × [illegible]	$\frac{2 [illegible] \times [illegible]}{[illegible]}$	$\frac{2 [illegible] \times [illegible]}{[illegible]}$	72$_{i-1}$ + 70 + 71 × [illegible]
1	1	53.03800×10⁶	1,063.7770×10⁶	−0.054016	−320,000	−0.003259026	−0.016205	78.55834×10⁶	−0.760142	−14.239858	−151.05072×10⁶	10,147.30240×10⁶	−0.270057	0.029722	88,424	−93,225	0.045023	−0.000307487	0.050256
	2	56.87200		−.057021			−.016205			−4.239858	−48.22584		−.289578	.009505					.046582
	3	36.60405		−.037341			−.016295			5.760142	42.23872		−.186087	−.008325					.042007
	4	49.82120		−.050740			−.016295			15.760142	157.03784		−.253677	−.030052					.030232
2	1	46.32300	1,739.9270	−0.053248	−180,000	−0.002069052	−0.026640	69.50708	−0.764980	−14.235020	−131.88433	8,995.79916	−0.266240	0.029618	83,520	−90,787	0.048007	−0.000448189	0.104048
	2	51.41725		−.059103			−.026640			−4.235020	−43.55062		−.295514	.009780					.096486
	3	32.40295		−.037350			−.026640			5.764980	37.40424		−.186740	−.008413					.088330
	4	43.75860		−.050209			−.026640			15.764980	137.97069		−.251407	−.030084					.080714
3	1	27.59630	1,027.0105	−0.053741	−80,000	−0.001557020	−0.034430	41.08042	−0.724610	−14.275390	−78.78959	5,330.04088	−0.268705	0.029564	52,307	−69,223	0.050931	−0.000510403	0.162900
	2	30.06095		−.058541			−.034430			−4.275390	−25.70446		−.292708	.009645					.149033
	3	18.58260		−.036188			−.034430			5.724610	21.27563		−.180039	−.007983					.136280
	4	26.46120		−.051505			−.034430			15.724610	83.21841		−.257653	−.031226					.122034
4	1	21.25785	812.6040	−0.052320	−20,000	−0.000402245	−0.036891	32.50416	−0.775328	−14.224672	−60.47577	4,134.40480	−0.261595	0.029254	43,844	−62,157	0.053955	−0.000601855	0.225409
	2	24.92600		−.061348			−.036891			−4.224672	−21.00083		−.306742	.010188					.206120
	3	14.50600		−.035852			−.036891			5.775328	16.82573		−.179262	−.008139					.186768
	4	20.51015		−.050480			−.036891			15.775328	64.71087		−.252400	−.031303					.167402

*For load problem:

$$\beta_{y,i}''=-\left(\frac{2}{EI_{yy}''}\right)_i\sum_{j=1}^{4}(u_{i,j,1}-u_{i-1,j,1})(AEz'')_{i,j,1}\qquad(\beta_{y,i}'')_0=\left(\frac{L}{EI_{yy}''}\right)_i(\bar{M}_y'')_i$$

**For temperature problem:

$$\delta u_i''=-\left(\frac{2}{AE}\right)_i\sum_{j=1}^{4}(u_{i,j,1}-u_{i-1,j,1})(AE)_{i,j,1}\qquad(\delta u_i'')_0=2\left(\frac{L}{AE}\right)_i\sum_{j=1}^{4}(AE\alpha T)_{i,j,1}$$

$$\beta_{z,i}''=-\left(\frac{2}{EI_{zz}''}\right)_i\sum_{j=1}^{4}(u_{i,j,1}-u_{i-1,j,1})(AEy'')_{i,j,1}\qquad(\beta_{z,i}'')_0=2\left(\frac{L}{EI_{zz}''}\right)_i\sum_{j=1}^{4}(AE\alpha Ty'')_{i,j,1}$$

Checks:

$$\sum_{j=1}^{4}(64)=\sum_{j=1}^{4}(67)=0$$

$$\sum_{j=1}^{4}[illegible]=-1$$

$$\sum_{j=1}^{4}[illegible]=-\frac{1}{5}$$

TABLE V.—STRINGER-DISPLACEMENT EQUATIONS

(a) Load problem *

(73)	(74)	(75)	(76)	(77)	(78)	(79)	(80)	(81)	(82)	(83)	(84)	(85)	(86)	(87)	(88)	(89)
i	j	$\sum S$	$u_{i-1,j-1}$	$u_{i-1,j}$	$u_{i-1,j+1}$	$u_{i,j-1}$	$u_{i,j+1}$	$u_{i+1,j-1}$	$u_{i+1,j}$	$u_{i+1,j+1}$	$\Delta w_i'$	$\Delta w_{i+1}'$	$\Delta\theta_i'$	$\Delta\theta_{i+1}'$	$\frac{\left(\frac{GtL}{h}\right)_{i,j,1}}{\sum S}$	$\frac{\left(\frac{GtL}{h}\right)_{i+1,j,1}}{\sum S}$
		**	$\frac{([illegible])_{i,j}}{(75)}$	$\frac{([illegible])_{i,j}-([illegible])_{i,j}}{(75)}+\frac{-([illegible])_{i,j+1}-2([illegible])_{i,j}}{(75)}$	$\frac{([illegible])_{i,j+1}}{(75)}$	$\frac{([illegible])_{i,j}+([illegible])_{i+1,j}}{(75)}$	$\frac{([illegible])_{i,j+1}+([illegible])_{i+1,j+1}}{(75)}$	$\frac{([illegible])_{i+1,j}}{(75)}$	$\frac{([illegible])_{i+1,j}-([illegible])_{i+1,j}}{(75)}+\frac{-([illegible])_{i+1,j+1}-2([illegible])_{i+1,j}}{(75)}$	$\frac{([illegible])_{i+1,j+1}}{(75)}$	$-\frac{([illegible])_{i,j}}{(75)}$	$-\frac{([illegible])_{i+1,j}}{(75)}$	$-\frac{([illegible])_{i,j}}{(75)}$	$-\frac{([illegible])_{i+1,j}}{(75)}$	$4\frac{([illegible])_{i,j}}{(75)}$	$4\frac{([illegible])_{i+1,j}}{(75)}$
1	1	1, 580, 926	0	0. 148536	0. 045276	0	0. 089976	0	0. 108473	0. 044700	−0. 070838	−0. 069922	−0. 657179	0. 652002	0. 283850	0. 279689
	2	1, 740, 895	. 041116	. 136564	. 041997	. 081708	. 083579	. 040592	. 107500	. 041583	−. 053503	−. 052837	. 127908	. 128254	. 214012	. 211348
	3	982, 648	. 074403	. 224435	. 074282	. 148072	. 148138	. 078669	. 183148	. 073856	0	0	. 000600	−. 000981	0	0
	4	1, 451, 952	. 050273	. 164978	0	. 100257	0	. 049984	. 124048	0	−. 063941	−. 063676	−. 869324	−. 863065	. 255764	. 254702
2	1	1, 175, 007	0	0. 145946	0. 060142	0	0. 089839	0	0. 112262	0. 029697	−0. 094078	−0. 046451	0. 877243	0. 430917	0. 376310	0. 185804
	2	1, 301, 596	. 054293	. 143783	. 055617	. 081101	. 083188	. 026809	. 108198	. 027521	−. 070670	−. 034216	. 171540	. 081071	. 282680	. 136865
	3	727, 614	. 099491	. 247336	. 099742	. 148722	. 149318	. 049231	. 156584	. 049576	0	0	−. 001258	−. 001725	0	0
	4	1, 086, 010	. 066826	. 165840	0	. 100041	0	. 033215	. 127330	0	−. 085132	−. 041556	−1. 153888	−. 562239	. 340528	. 166220
3	1	774, 605	0	0. 170292	0. 045047	0	0. 089460	0	0. 091092	0. 044412	−0. 070462	−0. 069463	0. 653662	0. 648157	0. 281847	0. 277850
	2	867, 291	. 040233	. 162871	. 041302	. 079899	. 082119	. 039666	. 105646	. 040817	−. 051851	−. 050636	. 121669	. 122248	. 205403	. 202543
	3	474, 609	. 075475	. 240057	. 076004	. 150052	. 151478	. 074588	. 156862	. 075475	0	0	−. 002644	−. 004435	0	0
	4	721, 733	. 049980	. 191597	0	. 099812	0	. 049632	. 110031	0	−. 062529	−. 062258	−. 846015	−. 839626	. 250115	. 249034
4	1	354, 588	0	0. 198992	0. 097020	0	0. 097020	0	0	0	−0. 151742	0	1. 415914	0	0. 606969	0
	2	406, 894	. 084548	. 225184	. 087001	. 084548	. 087001	0	0	0	−. 107930	0	. 260572	0	. 481719	0
	3	216, 890	. 163216	. 343252	. 165157	. 163216	. 165158	0	0	0	0	0	−. 009705	0	0	0
	4	330, 791	. 108289	. 240070	0	. 108289	0	0	0	0	−. 135838	0	−1. 881930	0	. 548352	0

$$^{*}u_{i,j,1}=\frac{1}{\sum S}\Bigg\{u_{i-1,j-1,1}\left(\frac{GtL}{4b}\right)_{i,j,\underline{1}}+u_{i,j-1,1}\left[\left(\frac{GtL}{4b}\right)_{i,j,\underline{1}}+\left(\frac{GtL}{4b}\right)_{i+1,j,\underline{1}}\right]+u_{i+1,j-1,1}\left(\frac{GtL}{4b}\right)_{i+1,j,\underline{1}}-\Delta\theta_i'\left[\left(\frac{Gtz'}{2}\right)_{i,j+1,\underline{1}}-\left(\frac{Gtz'}{2}\right)_{i,j,\underline{1}}+\left(\frac{Gty'}{2}\right)_{i,\underline{j},1}\right]+$$

$$u_{i-1,j,1}\left[\left(\frac{AE}{L}\right)_{i,j,1}-\left(\frac{GtL}{4b}\right)_{i,j,\underline{1}}-\left(\frac{GtL}{4b}\right)_{i,j+1,\underline{1}}-2\left(\frac{GtL}{4h}\right)_{i,\underline{j},1}\right]+u_{i+1,j,1}\left[\left(\frac{AE}{L}\right)_{i+1,j,1}-\left(\frac{GtL}{4b}\right)_{i+1,j,\underline{1}}-\left(\frac{GtL}{4b}\right)_{i+1,j+1,\underline{1}}-2\left(\frac{GtL}{4h}\right)_{i+1,\underline{j},1}\right]-\Delta w_i'\left(\frac{Gt}{2}\right)_{i,\underline{j},1}-\Delta w_{i+1}'\left(\frac{Gt}{2}\right)_{i+1,\underline{j},1}+$$

$$u_{i-1,j+1,1}\left(\frac{GtL}{4b}\right)_{i,j+1,\underline{1}}+u_{i,j+1,1}\left[\left(\frac{GtL}{4b}\right)_{i,j+1,\underline{1}}+\left(\frac{GtL}{4b}\right)_{i+1,j+1,\underline{1}}\right]+u_{i+1,j+1,1}\left(\frac{GtL}{4b}\right)_{i+1,j+1,\underline{1}}-\Delta\theta_{i+1}'\left[\left(\frac{Gtz'}{2}\right)_{i+1,j+1,\underline{1}}-\left(\frac{Gtz'}{2}\right)_{i+1,j,\underline{1}}+\left(\frac{Gty'}{2}\right)_{i+1,j,1}\right]\Bigg\}$$

$$^{**}\sum S=\left(\frac{AE}{L}\right)_{i,j,1}+\left(\frac{AE}{L}\right)_{i+1,j,1}+\left(\frac{GtL}{4b}\right)_{i,j,\underline{1}}+\left(\frac{GtL}{4b}\right)_{i,j+1,\underline{1}}+\left(\frac{GtL}{4b}\right)_{i+1,j,\underline{1}}+\left(\frac{GtL}{4b}\right)_{i+1,j+1,\underline{1}}+2\left(\frac{GtL}{4h}\right)_{i,\underline{j},1}+2\left(\frac{GtL}{4h}\right)_{i+1,j,1}$$

Check: (76)+(77)+(78)+(79)+(80)+(81)+(82)+(83)+(88)+(89)=1

TABLE V.—STRINGER-DISPLACEMENT EQUATIONS—Concluded

(b) Temperature problem *

90	91	92	93	94	95	96	97	98	99	100	101	102	103
i	j	$\sum S$	$u_{i-1,j-1}$	$u_{i-1,j}$	$u_{i-1,j+1}$	$u_{i,j-1}$	$u_{i,j+1}$	$u_{i+1,j-1}$	$u_{i+1,j}$	$u_{i+1,j+1}$	$\Delta v_i'$	$\Delta v_{i+1}'$	{c}
		**	$\frac{(20)_{i,j}}{(92)}$	$\frac{(9)_{i,j}-(20)_{i,j}-(20)_{i,j+1}}{(92)}$	$\frac{(20)_{i,j+1}}{(92)}$	$\frac{(20)_{i,j}+(20)_{i+1,j}}{(92)}$	$\frac{(20)_{i,j+1}+(20)_{i+1,j+1}}{(92)}$	$\frac{(20)_{i+1,j}}{(92)}$	$\frac{(9)_{i+1,j}-(20)_{i+1,j}-(20)_{i+1,j+1}}{(92)}$	$\frac{(20)_{i+1,j+1}}{(92)}$	$\frac{(42)_{i,j}}{(92)}$	$\frac{(42)_{i+1,j}}{(92)}$	$\frac{(10)_{i,j}-(10)_{i+1,j}}{(92)}$
1	1	1,135,864	0	0.403923	0.063016	0	0.125231	0	0.345615	0.062214	-0.063016	0.062214	0.001740
	2	1,370,641	.052222	.309366	.053341	.103780	.106157	.051558	.270760	.052815	.001119	.001258	.000104
	3	982,648	.074403	.224435	.074283	.148072	.148138	.073669	.183143	.073856	—.000120	.000186	.000818
	4	1,081,366	.067502	.393223	0	.134615	0	.067113	.337547	0	—.067502	—.067113	.001824
2	1	844,763	0	0.464713	0.083653	0	0.124959	0	0.285369	0.041306	0.083653	0.041306	0.011800
	2	1,028,556	.068705	.360812	.070381	.102630	.105208	.033925	.223512	.034826	.001676	.000898	.008304
	3	727,614	.099491	.247336	.099742	.148722	.149318	.049231	.156584	.049576	.000252	.000345	.007573
	4	810,844	.089504	.460163	0	.133991	0	.044487	.281854	0	—.080504	—.044487	.008892
3	1	557,833	0	0.432153	0.062553	0	0.124224	0	0.319400	0.061671	0.062553	0.061671	0.005595
	2	690,387	.050543	.332994	.051885	.100373	.103161	.049830	.259938	.051276	.001338	.001446	.002161
	3	474,609	.073475	.240067	.076004	.150062	.151478	.074588	.150862	.075475	.000529	.000887	.003045
	4	541,607	.066602	.421966	0	.132740	0	.066138	.312553	0	—.066602	—.066138	.004440
4	1	246,976	0	0.721414	0.139293	0	0.139293	0	0	0	0.139293	0	0.056192
	2	319,062	.107822	.562455	.110950	.107822	.110950	0	0	0	.003128	0	.041760
	3	216,890	.163217	.343252	.165158	.163216	.165157	0	0	0	.001941	0	.032731
	4	240,923	.148652	.702635	0	.148652	0	0	0	0	—.148652	0	.039610

$$^{*}u_{i,j,1}=\frac{1}{\sum S}\Bigg\{u_{i-1,j-1,1}\left(\frac{GtL}{4b}\right)_{i,j,\underline{1}}+u_{i,j-1,1}\left[\left(\frac{GtL}{4b}\right)_{i,j,\underline{1}}+\left(\frac{GtL}{4b}\right)_{i+1,j,\underline{1}}\right]+u_{i+1,j-1,1}\left(\frac{GtL}{4b}\right)_{i+1,j,\underline{1}}+u_{i-1,j,1}\left[\left(\frac{AE}{L}\right)_{i,j,1}-\left(\frac{GtL}{4b}\right)_{i,j,\underline{1}}-\left(\frac{GtL}{4b}\right)_{i,j+1,\underline{1}}\right]+\Delta v_i'\left[\left(\frac{Gt}{2}\right)_{i,j+1,\underline{1}}-\left(\frac{Gt}{2}\right)_{i,j,\underline{1}}\right]+$$

$$u_{i+1,j,1}\left[\left(\frac{AE}{L}\right)_{i+1,j,1}-\left(\frac{GtL}{4b}\right)_{i+1,j,\underline{1}}-\left(\frac{GtL}{4b}\right)_{i+1,j+1,\underline{1}}\right]+u_{i-1,j+1,1}\left(\frac{GtL}{4b}\right)_{i,j+1,\underline{1}}+u_{i,j+1,1}\left[\left(\frac{GtL}{4b}\right)_{i,j+1,\underline{1}}+\left(\frac{GtL}{4b}\right)_{i+1,j+1,\underline{1}}\right]+\Delta v_{i+1}'\left[\left(\frac{Gt}{2}\right)_{i+1,j+1,\underline{1}}-\left(\frac{Gt}{2}\right)_{i+1,j,\underline{1}}\right]+$$

$$u_{i+1,j+1,1}\left(\frac{GtL}{4b}\right)_{i+1,j+1,\underline{1}}+(AE\alpha T)_{i,j,1}-(AE\alpha T)_{i+1,j,1}\Bigg\}$$

$$^{**}\sum S=\left(\frac{AE}{L}\right)_{i,j,1}+\left(\frac{AE}{L}\right)_{i+1,j,1}+\left(\frac{GtL}{4b}\right)_{i,j,\underline{1}}+\left(\frac{GtL}{4b}\right)_{i,j+1,\underline{1}}+\left(\frac{GtL}{4b}\right)_{i+1,j,\underline{1}}+\left(\frac{GtL}{4b}\right)_{i+1,j+1,\underline{1}}$$

Check: (93)+(94)+(95)+(96)+(97)+(98)+(99)+(100)=1

TABLE VI.—MATRIX OF COEFFICIENTS

(a) Load problem

	u_{111}	u_{121}	u_{131}	u_{141}	u_{211}	u_{221}	u_{231}	u_{241}	u_{311}	u_{321}	u_{331}	u_{341}
u_{111}	0	0.081708	0	0	0.145946	0.054293	0	0	0	0	0	0
u_{121}	.089976	0	.148072	0	.060142	.143783	.099491	0	0	0	0	0
u_{131}	0	.083579	0	.100257	0	.055617	.247336	.066826	0	0	0	0
u_{141}	0	0	.148138	0	0	0	.099742	.165840	0	0	0	0
u_{211}	0.108473	0.040592	0	0	0	0.081101	0	0	0.170292	0.040233	0	0
u_{221}	.044700	.107500	.073669	0	.089839	0	.148722	0	.045047	.162371	.075475	0
u_{231}	0	.041583	.183143	.049984	0	.083138	0	.100041	0	.041302	.240057	.049980
u_{241}	0	0	.073856	.124043	0	0	.149318	0	0	0	.076004	.191597
u_{311}	0	0	0	0	0.112262	0.026809	0	0	0	0.079899	0	0
u_{321}	0	0	0	0	.029697	.108193	.049231	0	.069460	0	.150062	0
u_{331}	0	0	0	0	0	.027521	.156584	.033215	0	.062119	0	.099612
u_{341}	0	0	0	0	0	0	.049576	.127330	0	0	.151478	0
u_{411}	0	0	0	0	0	0	0	0	0.091092	0.039666	0	0
u_{421}	0	0	0	0	0	0	0	0	.044412	.105646	.074588	0
u_{431}	0	0	0	0	0	0	0	0	0	.040817	.156862	.049032
u_{441}	0	0	0	0	0	0	0	0	0	0	.075475	.110031
$\Delta w_1'$	−0.070838	−0.053503	0	−0.063941	0	0	0	0	0	0	0	0
$\Delta w_2'$	−.069922	−.052837	0	−.063676	−.094078	−.070670	0	−.065132	0	0	0	0
$\Delta w_3'$	0	0	0	0	−.046451	−.034216	0	−.041555	−.070462	−.051351	0	−.062529
$\Delta w_4'$	0	0	0	0	0	0	0	0	−.069463	−.060636	0	−.062258
$\Delta\theta_1'$	0.657179	0.127908	0.000600	−0.869324	0	0	0	0	0	0	0	0
$\Delta\theta_2'$	.652002	.128254	−.000931	−.863065	.877243	.171540	−.001258	−1.153883	0	0	0	0
$\Delta\theta_3'$	0	0	0	0	.430917	.081071	−.001725	−.562239	.653662	.121609	−.002644	−.846015
$\Delta\theta_4'$	0	0	0	0	0	0	0	0	.648157	.122248	−.004435	−.839626
c	0	0	0	0	0	0	0	0	0	0	0	0

	u_{411}	u_{421}	u_{431}	u_{441}	$\Delta w_1'$	$\Delta w_2'$	$\Delta w_3'$	$\Delta w_4'$	$\Delta\theta_1'$	$\Delta\theta_2'$	$\Delta\theta_3'$	$\Delta\theta_4'$
u_{111}	0	0	0	0	−0.751678	−0.749488	0	0	0.036202	0.036204	0	0
u_{121}	0	0	0	0	−.625182	−.623662	0	0	.007760	.007842	0	0
u_{131}	0	0	0	0	0	0	0	0	.000020	−.000032	0	0
u_{141}	0	0	0	0	−.623142	−.626850	0	0	−.043982	−.044014	0	0
u_{211}	0	0	0	0	0	−0.749488	−0.756768	0	0	0.036204	0.036210	0
u_{221}	0	0	0	0	0	−.623662	−.617504	0	0	.007842	.007546	0
u_{231}	0	0	0	0	0	0	0	0	0	−.000032	−.000090	0
u_{241}	0	0	0	0	0	−.626850	−.625728	0	0	−.044014	−.043666	0
u_{311}	0.198992	0.084548	0	0	0	0	−0.756768	−0.754346	0	0	0.036210	0.036220
u_{321}	.097020	.225184	.163216	0	0	0	−.617504	−.615690	0	0	.007546	.007648
u_{331}	0	.087001	.343252	.108289	0	0	0	0	0	0	−.000090	−.000142
u_{341}	0	0	.165157	.240070	0	0	−.625728	−.629962	0	0	−.043666	−.043716
u_{411}	0	0.084548	0	0	0	0	0	−0.754346	0	0	0	0.036220
u_{421}	.097020	0	.163216	0	0	0	0	−.615690	0	0	0	.007648
u_{431}	0	.087001	0	.108289	0	0	0	0	0	0	0	−.000142
u_{441}	0	0	.165158	0	0	0	0	−.629962	0	0	0	−.043716
$\Delta w_1'$	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta w_2'$	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta w_3'$	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta w_4'$	−.151742	−.107930	0	−.135838	0	0	0	0	0	0	0	0
$\Delta\theta_1'$	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta\theta_2'$	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta\theta_3'$	0	0	0	0	0	0	0	0	0	0	0	0
$\Delta\theta_4'$	1.415914	.260572	−.009705	−1.831930	0	0	0	0	0	0	0	0
c	0	0	0	0	0.026848	0.020340	0.027731	0.014020	−0.000345	−0.000266	−0.000354	−0.000182

TABLE VI.—MATRIX OF COEFFICIENTS—Concluded

(b) Temperature problem

	u_{111}	u_{121}	u_{131}	u_{141}	u_{211}	u_{221}	u_{231}	u_{241}	u_{311}	u_{321}
u_{111}	0	0.103780	0	0	0.464713	0.068705	0	0	0	0
u_{121}	.125231	0	.148072	0	.083653	.360812	.099491	0	0	0
u_{131}	0	.106157	0	.134615	0	.070381	.247336	.089504	0	0
u_{141}	0	0	.148138	0	0	0	.099742	.450163	0	0
u_{211}	0.345615	0.051558	0	0	0	0.102630	0	0	0.432153	0.050543
u_{221}	.062214	.270760	.073669	0	.124959	0	.148722	0	.062553	.332994
u_{231}	0	.052815	.183143	.067113	0	.105208	0	.133991	0	.051885
u_{241}	0	0	.073856	.337547	0	0	.149318	0	0	0
u_{311}	0	0	0	0	0.285369	0.033925	0	0	0	0.100373
u_{321}	0	0	0	0	.041306	.223512	.049231	0	.124224	0
u_{331}	0	0	0	0	0	.034826	.156584	.044487	0	.103161
u_{341}	0	0	0	0	0	0	.049576	.281854	0	0
u_{411}	0	0	0	0	0	0	0	0	0.319400	0.049530
u_{421}	0	0	0	0	0	0	0	0	.061671	.259938
u_{431}	0	0	0	0	0	0	0	0	0	.051276
u_{441}	0	0	0	0	0	0	0	0	0	0
$\Delta s_1'$	0.063016	0.001119	−0.000120	−0.067502	0	0	0	0	0	0
$\Delta s_2'$	.062214	.001258	.000186	−.067113	.083653	.001676	.000252	−.089504	0	0
$\Delta s_3'$	0	0	0	0	.041306	.000898	.000345	−.044487	.062553	.001338
$\Delta s_4'$	0	0	0	0	0	0	0	0	.061671	.001446
c	0.001740	0.000104	0.000818	0.001824	0.011800	0.008304	·0.007563	0.008892	0.005595	0.002161

	u_{331}	u_{341}	u_{411}	u_{421}	u_{431}	u_{441}	$\Delta s_1'$	$\Delta s_2'$	$\Delta s_3'$	$\Delta s_4'$
u_{111}	0	0	0	0	0	0	0.328816	0.327720	0	0
u_{121}	0	0	0	0	0	0	.007047	.007995	0	0
u_{131}	0	0	0	0	0	0	−.000542	.000849	0	0
u_{141}	0	0	0	0	0	0	−.335321	−.336564	0	0
u_{211}	0	0	0	0	0	0	0	0.327720	0.326763	0
u_{221}	.075475	0	0	0	0	0	0	.007995	.008681	0
u_{231}	.240057	.066602	0	0	0	0	0	.000849	.002350	0
u_{241}	.076004	.421966	0	0	0	0	0	−.336564	−.337794	0
u_{311}	0	0	0.721414	0.107822	0	0	0	0	0.326763	0.325706
u_{321}	.150062	0	.139293	.562455	.163217	0	0	0	.008681	.009449
u_{331}	0	.132740	0	.110950	.343252	.148682	0	0	.002350	.003986
u_{341}	.151478	0	0	0	.165158	.702635	0	0	−.337794	−.339140
u_{411}	0	0	0	0.107822	0	0	0	0	0	0.325706
u_{421}	.074588	0	.139293	0	.163216	0	0	0	0	.009449
u_{431}	.156862	.066138	0	.110950	0	.148682	0	0	0	.003986
u_{441}	.075475	.312553	0	0	.165157	0	0	0	0	−.339140
$\Delta s_1'$	0	0	0	0	0	0	0	0	0	0
$\Delta s_2'$	0	0	0	0	0	0	0	0	0	0
$\Delta s_3'$	.000529	−.066602	0	0	0	0	0	0	0	0
$\Delta s_4'$	.000887	−.066138	.139293	.003128	.001941	−.148682	0	0	0	0
c	0.003045	0.004440	0.056192	0.041760	0.032731	0.039610	0	0	0	0

TABLE VII.—CORRECTION CYCLE

(a) Load problem

	$\beta_{y,1}''$	$\beta_{y,2}''$	$\beta_{y,3}''$	$\beta_{y,4}''$
u_{111}	−0.054016	0.053248	----------	----------
u_{121}	−.057921	.059103	----------	----------
u_{131}	−.037341	.037350	----------	----------
u_{141}	−.050740	.050299	----------	----------
u_{211}	----------	−0.053248	0.053741	----------
u_{221}	----------	−.059103	.058541	----------
u_{231}	----------	−.037350	.036188	----------
u_{241}	----------	−.050299	.051505	----------
u_{311}	----------	----------	−0.053741	0.052320
u_{321}	----------	----------	−.058541	.061348
u_{331}	----------	----------	−.036188	.035852
u_{341}	----------	----------	−.051505	.050482
u_{411}	----------	----------	----------	−0.052320
u_{421}	----------	----------	----------	−.061348
u_{431}	----------	----------	----------	−.035852
u_{441}	----------	----------	----------	−.050480
$z_{t,1}''$	5	5	5	5
5th cycle	−35.25	−62.43	−85.11	−97.77
10th cycle	−.7006	−1.198	1.799	−2.156
14th cycle	.09770	.1949	.1669	.2673

(b) Temperature problem

	$\delta u_1''$	$\beta_{z,1}''$	$\delta u_2''$	$\beta_{z,2}''$	$\delta u_3''$	$\beta_{z,3}''$	$\delta u_4''$	$\beta_{z,4}''$
u_{111}	−0.270057	0.029722	0.266240	−0.029618	----------	----------	----------	----------
u_{121}	−.289578	.009505	.295514	−.009780	----------	----------	----------	----------
u_{131}	−.180687	−.008325	.186749	.008413	----------	----------	----------	----------
u_{141}	−.253677	−.030952	.251497	.030984	----------	----------	----------	----------
u_{211}	----------	----------	−0.266240	0.029618	0.268705	−0.029564	----------	----------
u_{221}	----------	----------	−.295514	.009780	.292703	−.009645	----------	----------
u_{231}	----------	----------	−.186749	−.008413	.180939	.007983	----------	----------
u_{241}	----------	----------	−.251497	−.030984	.257653	.031226	----------	----------
u_{311}	----------	----------	----------	----------	−0.268705	0.029564	0.261595	−0.029254
u_{321}	----------	----------	----------	----------	−.292703	.009645	.306742	−.010188
u_{331}	----------	----------	----------	----------	−.180939	−.007983	.179262	.008139
u_{341}	----------	----------	----------	----------	−.257653	−.031226	.252400	.031303
u_{411}	----------	----------	----------	----------	----------	----------	−0.261595	0.029254
u_{421}	----------	----------	----------	----------	----------	----------	−.306742	.010188
u_{431}	----------	----------	----------	----------	----------	----------	−.179262	−.008139
u_{441}	----------	----------	----------	----------	----------	----------	−.252400	−.031303
$y_{t,1}''$	----------	−14.239858	----------	−14.235020	----------	−14.275390	----------	−14.224672
$y_{t,2}''$	----------	−4.239858	----------	−4.235020	----------	−4.275390	----------	−4.224672
$y_{t,3}''$	----------	5.760142	----------	5.764980	----------	5.724610	----------	5.775328
$y_{t,4}''$	----------	15.760142	----------	15.764980	----------	15.724610	----------	15.775328
5th cycle	−83	1.951	−64	3.713	−94	5.256	−98	5.604
9th cycle	0	−.0199	0	.05887	−2	.07768	−2	.06513

TABLE VIII.—SUCCESSIVE VALUES OF DISPLACEMENT

(a) Load problem

	Initial values	1st cycle	Difference	2d cycle	3d cycle	4th cycle	5th cycle (correction)	6th cycle	7th cycle	8th cycle	9th cycle	10th cycle (correction)	11th cycle	12th cycle	13th cycle	14th cycle (correction)	Total value	Check cycle
u_{111}	−0.016295	−0.017584	-1289×10^{-6}	−1471	−1845	−2034	−2210	−2283	−2318	−2334	−2341	−2345	−2348	−2350	−2351	−2351	−0.018646	−0.018645
u_{121}	−.016295	−.016616	−321	−183	−309	−375	−551	−570	−579	−583	−585	−589	−590	−590	−590	−590	−.016885	−.016885
u_{131}	−.016295	−.013636	2659	3002	3407	3510	3334	3308	3428	3441	3447	3443	3447	3449	3449	3449	−.012846	−.012845
u_{141}	−.016295	−.016304	−9	419	621	705	620	588	618	631	637	633	637	639	640	640	−.015655	−.015654
u_{211}	−0.026640	−0.027979	−1339	−1710	−2280	−2543	−2855	−2960	−3012	−3036	−3048	−3054	−3060	−3062	−3064	−3063	−0.029703	−0.029703
u_{221}	−.026640	−.026813	−173	−19	−183	−257	−509	−509	−612	−618	−621	−627	−620	−629	−630	−629	−.027260	−.027260
u_{231}	−.026640	−.024009	2631	3750	4152	4312	4000	4094	4135	4154	4163	4157	4161	4163	4164	4165	−.022475	−.022475
u_{241}	−.026640	−.026748	−108	620	907	1051	730	831	875	895	904	898	904	905	907	908	−.025732	−.025732
u_{311}	−0.034480	−0.035657	−1177	−1574	−2101	−2345	−2771	−2890	−2945	−2971	−2983	−2992	−2999	−3002	−3003	−3002	−0.037432	−0.037433
u_{321}	−.034480	−.034687	−207	−36	−25	−107	−623	−649	−664	−670	−674	−683	−685	−685	−686	−685	−.035115	−.035115
u_{331}	−.034480	−.032184	2296	3480	3908	4100	3734	3837	3884	3905	3914	3905	3910	3912	3913	3914	−.030516	−.030515
u_{341}	−.034480	−.034428	2	738	1144	1324	898	990	1051	1074	1083	1074	1080	1082	1084	1085	−.033345	−.033344
u_{411}	−0.036891	−0.037801	−910	−1232	−1718	−1940	−2420	−2508	−2566	−2584	−2594	−2605	−2611	−2613	−2615	−2614	−0.039505	−0.039506
u_{421}	−.036891	−.036944	−53	140	67	30	−450	−508	−519	−525	−529	−540	−541	−541	−542	−541	−.037432	−.037432
u_{431}	−.036891	−.035236	1655	2685	3362	3475	2986	3094	3133	3140	3156	3145	3150	3151	3152	3153	−.033738	−.033738
u_{441}	−.036891	−.036936	−45	832	1104	1353	804	977	1021	1041	1049	1038	1042	1044	1046	1047	−.035844	−.035843
$\Delta w_1'$	0.059438	---	---	959	1103	---	1676	1706	1719	1729	---	1736	1737	1737	---	---	0.061175	0.061174
$\Delta w_2'$	.106210	---	---	1806	2441	---	3700	3769	3802	3817	---	3845	3847	3849	---	---	.110053	.110050
$\Delta w_3'$	.149871	---	---	1679	2100	---	3969	4053	4091	4109	---	4150	4155	4157	---	---	.154025	.154025
$\Delta w_4'$	.156662	---	---	1061	1382	---	3473	3540	3576	3594	---	3645	3650	3651	---	---	.160310	.160310
$\Delta\theta_1'$	−0.000345	---	---	−73	−96	---	−107	−113	−116	−117	---	−117	−118	−118	---	---	−0.000463	−0.000463
$\Delta\theta_2'$	−.000266	---	---	−103	−221	---	−248	−262	−268	−271	---	−273	−273	−274	---	---	−.000540	−.000540
$\Delta\theta_3'$	−.000354	---	---	−179	−250	---	−285	−302	−310	−314	---	−316	−317	−317	---	---	−.000671	−.000671
$\Delta\theta_4'$	−.000182	---	---	−170	−241	---	−275	−292	−300	−303	---	−305	−306	−307	---	---	−.000490	−.000490

(b) Temperature problem

	Initial values	1st cycle	Difference	2d cycle	3d cycle	4th cycle	5th cycle (correction)	6th cycle	7th cycle	8th cycle	9th cycle (correction)	Total value	Check cycle
u_{111}	0.050256	0.050710	454×10^{-6}	680	734	803	742	756	757	756	756	0.051012	0.051011
u_{121}	.046582	.046079	−503	−723	−810	−856	−897	−914	−917	−917	−917	.045665	.045664
u_{131}	.042907	.042735	−172	−345	−384	−412	−434	−435	−435	−435	−435	.042472	.042472
u_{141}	.039232	.039548	317	402	522	550	554	564	562	562	562	.039794	.039795
u_{211}	0.104643	0.105679	1036	1286	1544	1594	1477	1487	1487	1487	1486	0.106129	0.106129
u_{221}	.096486	.095376	−1110	−1386	−1594	−1664	−1744	−1757	−1759	−1759	−1759	.094727	.094727
u_{231}	.088330	.087856	−474	−723	−840	−852	−805	−805	−805	−805	−805	.087435	.087435
u_{241}	.080174	.080925	751	943	1080	1120	1115	1118	1118	1118	1119	.081293	.081292
u_{311}	0.162990	0.163540	550	1654	1740	1751	1582	1588	1589	1589	1586	0.164576	0.164577
u_{321}	.149633	.149052	−581	−1441	−1682	−1751	−1868	−1872	−1872	−1872	−1874	.147759	.147760
u_{331}	.136286	.136048	−238	−831	−885	−899	−963	−962	−961	−961	−963	.135323	.135323
u_{341}	.122934	.123322	388	1008	1107	1204	1193	1195	1196	1196	1195	.124129	.124130
u_{411}	0.225499	0.227888	2389	2986	3054	3080	2852	2856	2857	2857	2854	0.228353	0.228353
u_{421}	.206129	.203755	−2374	−2916	−3088	−3131	−3253	−3252	−3252	−3252	−3254	.202875	.202875
u_{431}	.186768	.185593	−1175	−1701	−1704	−1724	−1790	−1789	−1789	−1789	−1791	.184977	.184977
u_{441}	.167402	.169155	1753	2148	2160	2154	2144	2146	2147	2147	2146	.169548	.169548
$\Delta v_1'$	0.003675	---	---	54	61	---	52	54	54	---	---	0.003725	0.003728
$\Delta v_2'$	.011832	---	---	144	184	---	143	147	147	---	---	.011973	.011977
$\Delta v_3'$	.021508	---	---	214	270	---	184	188	188	---	---	.021695	.021695
$\Delta v_4'$	.032718	---	---	330	370	---	253	255	255	---	---	.032972	.032972

TABLE IX.—STRESSES

		Load problem*									Temperature problem**						Total				
(104)	(105)	(106)	(107)	(108)	(109)	(110)	(111)	(112)	(113)	(114)	(115)	(116)	(117)	(118)	(119)	(120)	(121)	(122)	(123)	(124)	(125)
i	j	$u_{i,j,1}$	$\Delta w_i'$	$\Delta\theta_i'$	$\overline{P}_{i,j,1}$	$\overline{\sigma}_{i,j,1}$	$\left(\frac{\tau L}{2}\right)_{i,j,\underline{1}}$	$\tau_{i,j,\underline{1}}$	$\left(\frac{\tau L}{2}\right)_{i,\underline{j},1}$	$\tau_{i,\underline{j},1}$	$u_{i,j,1}$	$\Delta v_i'$	$\overline{P}_{i,j,1}$	$\overline{\sigma}_{i,j,1}$	$\left(\frac{\tau L}{2}\right)_{i,j,\underline{1}}$	$\tau_{i,j,\underline{1}}$	$\overline{\sigma}_{i,j,1}$	$\overline{\sigma}_{i,j,8}$	$\tau_{i,j,\underline{1}}$	$\tau_{i,j,\underline{8}}$	$\tau_{i,\underline{j},1}$
					(9)[(106)$_{i,j}$−(106)$_{i-1,j}$]	(109)/(7)	(20)[(106)$_{i,j}$−(106)$_{i,j-1}$+(106)$_{i-1,j}$−(106)$_{i-1,j-1}$]−(108)(43)	(111)(21)	2×(28)[(106)$_{i,j}$+(106)$_{i-1,j}$]+(107)(26)+(108)(48)	(113)(29)			(9)[(115)$_{i,j}$−(115)$_{i-1,j}$]−(10)	(117)/(7)	(20)[(115)$_{i,j}$−(115)$_{i,j-1}$+(115)$_{i-1,j}$−(115)$_{i-1,j-1}$]+(18)(116)	(119)(21)	(118)+(110)	(118)−(110)	(120)+(112)	(120)−(112)	(114)
1	1	−0.018645	0.061174	−0.000468	−9,889	−8,909	----------	------	3,321	5,189	0.051011	0.003728	−1,888	−1,701	----------	------	−10,610	7,208	------	------	5,189
	2	−.016885			−9,603	−8,350	292	730	2,659	5,214	.045664		2,470	2,148	−116	−290	−6,202	10,498	440	−1,020	5,214
	3	−.012845			−4,710	−6,865	465	1,163	----------	------	.042472		721	974	39	98	−5,891	7,339	1,261	−1,065	------
	4	−.015654			−7,799	−7,722	−36	−90	2,019	3,959	.039795		−1,304	−1,291	77	193	−9,013	6,431	103	283	3,959
2	1	−0.029703	0.110056	−0.000540	−5,122	−5,227	----------	------	2,224	3,322	0.106129	0.011977	−1,484	−1,463	----------	------	−6,690	3,764	------	------	3,322
	2	−.027269			−5,339	−5,085	487	1,218	2,126	4,165	.094727		1,870	1,781	−337	−843	−3,304	6,866	375	−2,060	4,165
	3	−.022475			−3,129	−4,741	835	2,088	----------	------	.087485		563	853	108	270	−3,888	5,594	2,358	−1,818	------
	4	−.025732			−4,410	−4,955	−244	−610	1,650	3,235	.081292		−999	−1,122	229	573	−6,077	3,833	−37	1,183	3,235
3	1	−0.037433	0.154025	−0.000671	−2,133	−3,615	----------	------	1,535	4,797	0.164577	0.021695	−870	−1,475	----------	------	−5,090	2,140	------	------	4,797
	2	−.035115			−2,359	−3,805	283	1,415	1,877	5,508	.147760		1,126	1,816	−228	−1,140	−1,989	5,621	275	−2,555	5,508
	3	−.030515			−1,494	−3,934	457	2,285	----------	------	.135323		355	934	70	350	−3,000	4,868	2,635	−1,935	------
	4	−.033344			−2,014	−3,730	−97	−485	1,088	4,352	.124130		−612	−1,133	157	785	−4,863	2,597	300	1,270	4,352
4	1	−0.039506	0.160810	−0.000490	−441	−959	----------	------	676	2,112	0.228353	0.032972	−321	−698	----------	------	−1,657	261	------	------	2,112
	2	−.037432			−578	−1,112	235	1,175	723	2,892	.202875		414	796	−321	−1,605	−316	1,908	−430	−2,780	2,892
	3	−.033738			−469	−1,563	380	1,900	----------	------	.184977		134	447	93	465	−1,116	2,010	2,365	−1,485	------
	4	−.035843			−513	−1,221	−89	−445	601	2,404	.169548		−227	−540	227	1,135	−1,761	681	690	1,580	2,404

$$^{*}\left(\frac{\tau t L}{2}\right)_{i,j,\underline{k}}=\left(\frac{GtL}{4b}\right)_{i,j,\underline{k}}(u_{i,j,k}+u_{i-1,j,k}-u_{i,j-1,k}-u_{i-1,j-1,k})-\left(\frac{Gtz'}{2}\right)_{i,j,\underline{k}}\Delta\theta_i'$$

$$\left(\frac{\tau t L}{2}\right)_{i,\underline{j},k}=2\left(\frac{GtL}{4h}\right)_{i,\underline{j},k}(u_{i,j,k}+u_{i-1,j,k})+\left(\frac{Gt}{2}\right)_{i,\underline{j},k}\Delta w_i'+\left(\frac{Gty'}{2}\right)_{i,\underline{j},k}\Delta\theta_i'$$

$$\overline{P}_{i,j,k}=\left(\frac{AE}{L}\right)_{i,j,k}(u_{i,j,k}-u_{i-1,j,k})$$

Checks:

(109)$_{i,j}$−(109)$_{i+1,j}$=(111)$_{i+1,j+1}$−(111)$_{i+1,j}$+(111)$_{i,j+1}$−(111)$_{i,j}$−(113)$_{i+1,j}$−(113)$_{i,j}$

$10\sum_{j=1}^{4}$ (109)$_{i,j}$=(56) $\sum_{j=1}^{4}$ (113)$_{i,j}$=(39)

$10\sum_{j=1}^{4}$ (111)$_{i,j}$−$\sum_{j=1}^{4}$ (113)(33)$_{i,j}$=−(40)

$$^{**}\left(\frac{\tau t L}{2}\right)_{i,j,\underline{k}}=\left(\frac{GtL}{4b}\right)_{i,j,\underline{k}}(u_{i,j,k}+u_{i-1,j,k}-u_{i,j-1,k}-u_{i-1,j-1,k})+\left(\frac{Gt}{2}\right)_{i,j,\underline{k}}\Delta v_i'$$

$$\overline{P}_{i,j,k}=\left(\frac{AE}{L}\right)_{i,j,k}(u_{i,j,k}-u_{i-1,j,k})-(AE\alpha T)_{i,j,k}$$

Checks:

(117)$_{i,j}$−(117)$_{i+1,j}$=(119)$_{i+1,j+1}$−(119)$_{i+1,j}$+(119)$_{i,j+1}$−(119)$_{i,j}$

$\sum_{j=1}^{4}$ (117)$_{i,j}$=0 $\sum_{j=1}^{4}$ (117)(22)$_{i,j}$=0 $\sum_{j=1}^{4}$ (119)$_{i,j}$=0